Quantization of geometrical structures in locally covariant field theory

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Outline of the talk

1. Introduction
   - Effective quantum gravity
   - Local covariance

2. BV formalism for gravity
   - Kinematical structure
   - Dynamics and symmetries
   - BV complex

3. Quantization
   - Deformation quantization
   - Applications
Problems with quantum gravity

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- "Points" lose their meaning.
- It is not clear what should be an observable.
- Need for "background independance".
- What replaces the classical spacetime structure in Planck scale?
Objectives of our program

Apply the methods of locally covariant quantum field theory to understand some of the features of quantum gravity.

Formulate perturbative quantum gravity as an effective theory that is valid in a given physical situation.

Answer some interpretational questions.

Find a relation to experiment: QG corrections to some processes, black hole radiation, cosmology.

Understand the small scale structure of spacetime: relation to noncommutative geometry.
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Diffeomorphism transformation: move our experimental setup to a different region $\mathcal{O}'$. 

$\Phi(\mathcal{O}, g)(f)[h]$
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- A good language to formalize it is the category theory. We need following categories:

  - $\text{Loc}$ where the objects are all four-dimensional, globally hyperbolic oriented and time-oriented spacetimes $M = (M, g)$. Morphisms: isometric embeddings preserving orientation, time-orientation and the causal structure.
  - $\text{Vec}$ with (small) topological vector spaces as objects and injective continuous homomorphisms of topological vector spaces as morphisms.

(Katarzyna Rejzner  QG in LCFT 5 / 24)
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We define a contravariant functor \( \mathcal{E} : \text{Loc} \to \text{Vec} \), which assigns to a spacetime the corresponding configuration space and acts on morphisms \( \chi : \mathcal{M} \to \mathcal{N} \) as \( \mathcal{E}\chi = \chi^* : \mathcal{E}(\mathcal{N}) \to \mathcal{E}(\mathcal{M}) \).
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In a similar way we define a covariant functor $\mathcal{E}_c : \text{Loc} \rightarrow \text{Vec}$ by setting $\mathcal{E}\chi = \chi_*$, where:

$$
\chi_* h = \begin{cases} 
(\chi^{-1})^* h(x) , & x \in \chi(M), \\
0 , & \text{else}
\end{cases}
$$
We consider the space of smooth functionals on $\mathcal{E}(\mathcal{M})$, i.e. $C^\infty(\mathcal{E}(\mathcal{M}), \mathbb{R})$. 
Functionals

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- The support of $F \in C^\infty(\mathcal{E}(\mathcal{M}), \mathbb{R})$ is defined as:

$$\text{supp } F = \{ x \in M | \forall \text{ neighbourhoods } U \text{ of } x \exists h_1, h_2 \in \mathcal{E}(\mathcal{M}), \text{ supp } h_2 \subset U \text{ such that } F(h_1 + h_2) \neq F(h_1) \}.$$
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- $F$ is local if it is of the form: $F(h) = \int_M f(j_x(h))(x)$, where $f$ is a density-valued function on the jet bundle over $M$ and $j_x(h)$ is the jet of $\varphi$ at the point $x$. 

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- \( \mathcal{F}(\mathcal{M}) \doteq \text{the space of multilocal functionals (products of local)} \).
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- We restrict ourselves to smooth maps $X$ with image in $E_c(M)$. They act on $\mathfrak{F}(\mathcal{M})$ as derivations: $\partial_X F(h) := \langle F^{(1)}(h), X(h) \rangle$
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- We consider only the multilocal (products of local vector fields and local functionals) vector fields with compact spacetime support.
- $\mathcal{V}(\mathcal{M})$ = the space of vector fields with above properties.
- $\mathcal{V}$ becomes a (covariant) functor after setting: $\mathcal{V}\chi(X) = \mathcal{E}_c\chi \circ X \circ \mathcal{E}\chi$. 

Vector fields
Fields as natural transformations

In the framework of locally covariant field theory [Brunetti-Fredenhagen-Verch 2003] fields are natural transformation between certain functors. For the sake of this talk let $\Phi \in \text{Nat}(\mathcal{D}, \mathcal{F})$, where $\mathcal{D}$ is the functor of test function spaces $\mathcal{D}(\mathcal{M}) = \mathcal{C}_c^\infty(\mathcal{M})$ (one could substitute $\mathcal{F}$ with a functor to the category of Poisson or $C^*$ algebras).
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- The condition for $\Phi$ to be a natural transformation: $\Phi_\mathcal{O}(f)[\chi^* h] = \Phi_\mathcal{M}(\chi^* f)[h]$.

- In classical gravity we understand physical quantities not as pointwise objects but rather as something defined on all the spacetimes in a coherent way.
The dynamics is introduced by a generalized Lagrangian $L$ which is a natural transformation between functors $\mathcal{D}$ and $\mathcal{F}_{\text{loc}}$. The action $S(L)$ is an equivalence class of Lagrangians, where $L_1 \sim L_2$ if $\text{supp}(L_1,\mathcal{M} - L_2,\mathcal{M})(f) \subset \text{supp}df \forall f \in \mathcal{D}(\mathcal{M})$. 
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- For GR: $L_{(M,g)}(f)[h] \overset{\dagger}{=} \int R[\tilde{g}]f \ d \text{vol}_{(M,\tilde{g})}, \quad \tilde{g} = g + h$. 


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The E-L derivative of $S(L)$ is a natural transformation $S' : \mathcal{E} \rightarrow \mathcal{E}'$ defined as $\langle S'_M(h_0), h \rangle = \langle L_M(f)^{(1)}(h_0), h \rangle$, where $f \equiv 1$ on supp $h$. The field equation is: $S'_M(h_0) = 0$. The space of solutions is denoted by $\mathcal{E}_S(\mathcal{M})$. 
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A symmetry of $S$ is a direction in $\mathcal{E}(\mathcal{M})$ in which the action is constant, i.e. it is a vector field $X \in \mathcal{V}(\mathcal{M})$ such that $\forall h_0 \in \mathcal{E}(\mathcal{M})$: $0 = \langle S'_\mathcal{M}(h_0), X(h_0) \rangle$. 
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diffeomorphisms, i.e. vector fields in $\mathfrak{X}(\mathcal{M}) \doteq \Gamma(TM)$. This
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We can now define a Lie algebra $\mathfrak{X}$, which provides us with a
notion of transforming all the spacetimes in a coherent way:

$$
\mathfrak{X} \doteq \prod_{\mathcal{M} \in \text{Obj}(\text{Loc})} \mathfrak{X}(\mathcal{M})
$$
Diffeomorphism invariance

- Let $\vec{\xi} \in \mathcal{X}$ with all the components compactly supported and $\alpha_\mathcal{M} = \exp(\xi_\mathcal{M})$ a family of diffeomorphisms constructed via the exponential mapping. The action of diffeomorphisms on natural transformations is given by:

$$(\bar{\alpha}\Phi)_{(M,g)}(f)[h] = \Phi_{(M,g)}(\alpha^{-1}_M f)[\alpha^*_M \tilde{g} - g].$$

- The derived action reads:

$$\left(\bar{\xi}\Phi\right)_{(M,g)}(f)[h] =$$

$$\left\langle (\Phi_{(M,g)}(f))^{(1)}(h), \mathcal{L}_{\xi_\mathcal{M}} \tilde{g} \right\rangle + \Phi_{(M,g)}(\mathcal{L}_{\xi_\mathcal{M}} f)[h]$$

- The right hand side is well defined also if we drop the compact support condition on $\vec{\xi}$, so we can adapt the above formula as the definition of the action of $\mathcal{X}$ on $\text{Nat}(\mathcal{D}, \mathcal{F})$. 
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- Diffeomorphism invariance is now the statement that: $\tilde{\xi}\Phi = 0$.
- Example: $\int R[\tilde{g}] f \, d \text{vol}_{(M, \tilde{g})}$ is invariant, but $\int R[\tilde{g}] f \, d \text{vol}_{(M, g)}$ is not.
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We identify \( \mathcal{E}_S(M) \) with its algebra of functions \( \mathcal{F}_S(M) \) and characterize it by its Koszul resolution (see [Costello 2011] for a finite dimensional version).
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$$\mathcal{F}_S(\mathcal{M}) = H_0\left( \bigwedge \mathfrak{D}(\mathcal{M}), S'_M(.) \right).$$

The underlying algebra of this differential complex is a certain completion of the odd cotangent bundle $\Pi T^* \mathcal{E}(\mathcal{M})$ of $\mathcal{E}(\mathcal{M})$. 
To incorporate the gauge invariance we replace the original configuration space $\mathcal{E}(\mathcal{M})$ with a graded manifold $\overline{\mathcal{E}}(\mathcal{M}) \doteq \mathcal{E}(\mathcal{M}) \oplus \mathfrak{X}(\mathcal{M})$ characterized by it’s algebra of functions $\mathcal{F}(\mathcal{M}) \hat{\otimes} \bigwedge \mathfrak{X}'(\mathcal{M}) = C^\infty_m \left( \mathcal{E}(\mathcal{M}), \bigwedge \mathcal{V}(\mathcal{M}) \right)$.

The underlying algebra of the Koszu resolution is the odd cotangent bundle $\Pi T^* \overline{\mathcal{E}}$ of $\overline{\mathcal{E}}$ and taking into account regularity conditions and topological completion we obtain the BV complex:

$$\mathcal{BV}(\mathcal{M}) = C^\infty_m \left( \mathcal{E}(\mathcal{M}), \bigwedge \mathcal{E}_c(\mathcal{M}) \hat{\otimes} \bigwedge \mathfrak{X}'(\mathcal{M}) \hat{\otimes} S^* \mathfrak{X}_c(\mathcal{M}) \right)$$
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Antifields: derivations of $\mathcal{F}(\mathcal{M})$, $\#_{af} = 1$, $\#_{gh} = -1$
To incorporate the gauge invariance we replace the original configuration space $\mathcal{E}(\mathcal{M})$ with a graded manifold $\overline{\mathcal{E}}(\mathcal{M}) = \mathcal{E}(\mathcal{M}) \oplus \mathcal{X}(\mathcal{M})$ characterized by it’s algebra of functions $\mathcal{F}(\mathcal{M}) \wedge \mathcal{X}'(\mathcal{M}) = C_{\text{ml}}(\mathcal{E}(\mathcal{M}), \Lambda \mathcal{Y}(\mathcal{M}))$.

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- The assignment of the BV complex to the manifold is a functor from $\text{Loc}$ to the category of graded algebras.
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$$Fld = \bigoplus_{k=0}^{\infty} \text{Nat}(\mathcal{E}_c^k, \mathcal{BV}),$$

where $\mathcal{E}_c^k$ be a functor from the category $\text{Loc}$ to the product category $\text{Vec}^k$, that assigns to a spacetime $\mathcal{M}$ a $k$-fold product of the test section spaces $\mathcal{E}_c(\mathcal{M}) \times \ldots \times \mathcal{E}_c(\mathcal{M})$. 
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The set $\text{Fld}$ becomes a graded algebra if we set:

$$\Phi \Psi \big|_{\mathcal{M}} (f_1, \ldots, f_{p+q}) =$$

$$= \frac{1}{p! q!} \sum_{\pi \in P_{p+q}} \Phi \big|_{\mathcal{M}} (f_{\pi(1)}, \ldots, f_{\pi(p)}) \Psi \big|_{\mathcal{M}} (f_{\pi(p+1)}, \ldots, f_{\pi(p+q)}) .$$

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The BV-differential on $Fld$ is given by:

$$(s\Phi)(\mathcal{M})(f) = \{\Phi_{\mathcal{M}}(f), S + \gamma\} + \Phi_{\mathcal{M}}(\mathcal{L}_C f),$$

where $C \in \mathcal{X}(\mathcal{M})$ is the ghost and $\gamma$ is the Chevalley-Eilenberg differential, which acts on $Fld$ via infinitesimal diffeomorphism transformations along the ghost fields $C$. For $\Phi \in \text{Nat}(\mathcal{E}_c, \mathcal{F})$:

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- For example: $\Phi_{(M, g)}(f)(h) = \int_M R_{\mu\nu\alpha\beta}[\tilde{g}] R^{\mu\nu\alpha\beta}[\tilde{g}] f d\text{vol}(M, \tilde{g}).$
Gauge fixing

- Gauge fixing is implemented by means of the so called gauge fixing fermion $\Psi \in Fld$ with ghost number $\#gh = 1$. 
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- We define an automorphism of $\mathcal{BV}(\mathcal{M})$ by

$$\alpha_\Psi(X) := \sum_{n=0}^{\infty} \frac{1}{n!} \{\Psi, \mathcal{M}(f), \ldots, \{\Psi, \mathcal{M}(f), X\} \ldots \},$$

where $f \equiv 1$ on the support of $X$. This automorphism in a simple way extends to $Fld$. 
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- It holds: $H^0(s^\Psi, \alpha_\Psi(Flb)) = H^0(s, Flb) = Flb_{\text{inv}}$. 
Equations of motion and Poisson bracket

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$$\{ F, G \}_0^g = \left\langle F^{(1)}, \Delta_g G^{(1)} \right\rangle$$

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- This Poisson structure can be naturally extended to a Poisson bracket $\{.,.\}_0$ on $Fld$. 
We start with the deformation quantization of \((\text{Fld}, \{., .\}_0)\).
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We need to include into the space of functionals on \(\mathcal{E}(\mathcal{M})\) some more singular objects. The right notion of regularity is related to a certain wavefront set property of Hadamard 2-point functions (microlocal spectrum condition). The resulting space will be denoted by \(\mathcal{BV}_{\mu c}(\mathcal{M})\).
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The deformation quantization of \((\mathcal{V}_\mu_{0c}(\mathcal{M}), \{.,.\}_0^g)\) can be performed in the standard way, by introducing a \(\star\)-product:

\[
(F \star_{H} G) \doteq m \circ \exp(i\hbar \Gamma \omega_H)(F \otimes G),
\]

where \(\Gamma \omega_H \doteq \frac{1}{2} \int dx \, dy \omega_H(x, y) \frac{\delta}{\delta \varphi(x)} \otimes \frac{\delta}{\delta \varphi(y)}\) and 

\[\omega_H = \frac{i}{2} \Delta g + H\] is the Hadamard 2-point function (satisfies the linearized EOM’s in both arguments and the \(\muSC\)).
With some technical considerations the deformation quantization on each $M$ leads to a deformation quantization on the space of fields and we obtain $\{Fl_{\mu c}[\hbar, \lambda], \star\}$. 
Interaction

- With some technical considerations the deformation quantization on each $\mathcal{M}$ leads to a deformation quantization on the space of fields and we obtain $\{Fld_{\mu c}[[\hbar, \lambda]], \star\}$.
- In the next step we have to introduce the interaction, i.e. consider the algebras $\{\mathcal{B}\mathcal{V}_{\mu c}(\mathcal{M})[[\hbar, \lambda]], \star_H\}$ and define on them the renormalized time-ordered products $\cdot_T$ by the Epstein-Glaser method.
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- Time ordered products on different $\mathcal{M}$ can be defined in a covariant way, which allows to extend it to a product on $\text{Fld}_{\mu c}[[\hbar, \lambda]]$. 

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- Time ordered products on different $\mathcal{M}$ can be defined in a covariant way, which allows to extend it to a product on $\text{Fld}_{\mu c}[[\hbar, \lambda]]$.
- One can define the formal S-matrix as: $\mathcal{S}(V^g) \doteq e_{\mathcal{T}}^{V^g}$.
- Interacting fields are obtained from free ones in $\{\text{Fld}_{\mu c}[[\hbar, \lambda]], \ast\}$ by the Bogoliubov formula:

$$ (R_V(\Phi))_\mathcal{M}(f) \doteq \left. \frac{d}{d\lambda} \right|_{\lambda=0} \mathcal{S}(V^g)^{-1} \ast \mathcal{S}(V^g + \lambda \Phi_\mathcal{M}(f)) . $$
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The BV construction can be applied the algebra of fields $Fld$ and gives a homological interpretation to the notion of gauge invariant physical quantities in general relativity.
Quantized geometrical structures

- In general relativity the basic physical objects are fields (natural transformations), since they are defined not on a fixed background but rather on a class of spacetimes in a coherent way.

- The BV construction can be applied the algebra of fields $Fld$ and gives a homological interpretation to the notion of gauge invariant physical quantities in general relativity.

- The algebra $Fld_{μc}$ can be equipped with the noncommutative $⋆$-product, which provides the deformation quantization of the free theory. The interaction is next introduced in the perturbative way and we obtain a notion of interacting quantum fields $R_V(Φ)$, where $Φ$ is a classical field constructed covariantly from the metric. For example: $\int R[\tilde{g}]f \ d \text{vol}(M,\tilde{g})$. 
Although we parametrize $Fld_{\mu c}$ with spacetimes, the construction is fully covariant and physical quantities are invariant under reparametrization.
Physical interpretation

- Although we parametrize $Fld_{\mu c}$ with spacetimes, the construction is fully covariant and physical quantities are invariant under reparametrization.

- The background independence would mean that the algebraic structure on $Fld_{\mu c}$ doesn’t depend on the split into the free and interacting part. This can be obtained as a certain renormalization condition called perturbative agreement ([Hollands, Wald, 2004] for the scalar field). Work in progress.
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- The physical interpretation of the theory is provided by constructing states on $\{Fld_{\mu c}[[\hbar, \lambda]], \star\}$. This problem is not entirely solved, since one needs to prove the existence of “gauge invariant” states on arbitrary $\mathcal{M}$ for the linearized theory. Up to now states can be explicitly given only on some special classes of spacetimes (for example ultrastatic).
Spacetime in Planck scale

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- The principle of gravitational stability against localization of events was proposed in [Doplicher, Fredenhagen, Roberts, CMP 95]. It states that a physical reason for Planck scale noncommutativity of spacetime is the fact that we cannot measure all the spacetime coordinates with arbitrary precision, because this would result in forming a trapped surface. This principle was used in [DFR 95] to derive the STUR in a simplified model.
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- In a recent work of [Doplicher, Morsella, Pinamonti 2012] this problem is studied for a model of scalar field coupled to gravity in the semiclassical approximation. We hope that using our notions of quantized metric and curvature one can move this one step further and include quantum gravity corrections.
Thank you for your attention