Lokalnie kowariantna kwantowa teoria pola jako podejście do kwantowej grawitacji

Katarzyna Rejzner

II. Institute for Theoretical Physics, Hamburg University

Kraków, 08.01.2010

Outline of the talk

Problems with quantum gravity

- Spacetime is dynamical
- "Points" lost their meaning
- It is not clear what should be an observable
- "background independance"
- Renormalizability

Problems with quantum gravity

- Spacetime is dynamical
- "Points" lost their meaning
- It is not clear what should be an observable
- "background independance"
- Renormalizability

Problems with quantum gravity

- Spacetime is dynamical
- "Points" lost their meaning
- It is not clear what should be an observable
- "background independance"
- Renormalizability

Problems with quantum gravity

- Spacetime is dynamical
- "Points" lost their meaning
- It is not clear what should be an observable
- "background independance"
- Renormalizability

Problems with quantum gravity

- Spacetime is dynamical
- "Points" lost their meaning
- It is not clear what should be an observable
- "background independance"
- Renormalizability

Problems with quantum gravity

- Spacetime is dynamical
- "Points" lost their meaning
- It is not clear what should be an observable
- "background independance"
- Renormalizability

Problems with quantum gravity

- Spacetime is dynamical
- "Points" lost their meaning
- It is not clear what should be an observable
- "background independance"
- Renormalizability

Problems with quantum gravity

- Spacetime is dynamical
- "Points" lost their meaning
- It is not clear what should be an observable
- "background independance"
- Renormalizability

Objectives

- formulate a consistent theory that is valid in a given physical situation
- answer some interpretational questions
- find a relation to experiment
- understand better problems of other approaches

Objectives

- formulate a consistent theory that is valid in a given physical situation
- answer some interpretational questions
- find a relation to experiment
- understand better problems of other approaches

Objectives

- formulate a consistent theory that is valid in a given physical situation
- answer some interpretational questions
- find a relation to experiment
- understand better problems of other approaches

Objectives

- formulate a consistent theory that is valid in a given physical situation
- answer some interpretational questions
- find a relation to experiment
- understand better problems of other approaches

Category

A category $\mathfrak{C}$ consists of

- a class $Obj(\mathfrak{C})$ of objects,
- a class $hom(\mathfrak{C})$ of morphisms between the objects. Each morphism f has a unique source object a and target object b where $a, b \in Obj(\mathfrak{C})$.
  Notation: if $f : a \to b$ then we write $f \in hom(a, b)$
- for $a, b, c \in Obj(\mathfrak{C})$, a binary operation $hom(a, b) \times hom(b, c) \to hom(a, c)$ called composition of morphisms. Notation: $f \circ g$.

such that the following axioms hold:

- associativity if $f : a \to b$, $g : b \to c$ and $h : c \to d$ then $h \circ (g \circ f) = (h \circ g) \circ f$
- identity for every object c, there exists a morphism $\mathrm{id}_c : c \to c$, such that for every $hom(a, b) \ni f$ we have: $\mathrm{id}_b \circ f = f \circ \mathrm{id}_a$=f.

Category

A category $\mathfrak{C}$ consists of

- a class $Obj(\mathfrak{C})$ of objects,
- a class $hom(\mathfrak{C})$ of morphisms between the objects. Each morphism f has a unique source object a and target object b where $a, b \in Obj(\mathfrak{C})$.
  Notation: if $f : a \to b$ then we write $f \in hom(a, b)$
- for $a, b, c \in Obj(\mathfrak{C})$, a binary operation $hom(a, b) \times hom(b, c) \to hom(a, c)$ called composition of morphisms. Notation: $f \circ g$.

such that the following axioms hold:

- associativity if $f : a \to b$, $g : b \to c$ and $h : c \to d$ then $h \circ (g \circ f) = (h \circ g) \circ f$
- identity for every object c, there exists a morphism $\mathrm{id}_c : c \to c$, such that for every $hom(a, b) \ni f$ we have: $\mathrm{id}_b \circ f = f \circ \mathrm{id}_a$=f.

Category

A category $\mathfrak{C}$ consists of

- a class $Obj(\mathfrak{C})$ of objects,
- a class $hom(\mathfrak{C})$ of morphisms between the objects. Each morphism f has a unique source object a and target object b where $a, b \in Obj(\mathfrak{C})$.
  Notation: if $f : a \to b$ then we write $f \in hom(a, b)$
- for $a, b, c \in Obj(\mathfrak{C})$, a binary operation $hom(a, b) \times hom(b, c) \to hom(a, c)$ called composition of morphisms. Notation: $f \circ g$.

such that the following axioms hold:

- associativity if $f : a \to b$, $g : b \to c$ and $h : c \to d$ then $h \circ (g \circ f) = (h \circ g) \circ f$
- identity for every object c, there exists a morphism $\mathrm{id}_c : c \to c$, such that for every $hom(a, b) \ni f$ we have: $\mathrm{id}_b \circ f = f \circ \mathrm{id}_a$=f.

Category

A category $\mathfrak{C}$ consists of

- a class $Obj(\mathfrak{C})$ of objects,
- a class $hom(\mathfrak{C})$ of morphisms between the objects. Each morphism f has a unique source object a and target object b where $a, b \in Obj(\mathfrak{C})$.
  Notation: if $f : a \to b$ then we write $f \in hom(a, b)$
- for $a, b, c \in Obj(\mathfrak{C})$, a binary operation $hom(a, b) \times hom(b, c) \to hom(a, c)$ called composition of morphisms. Notation: $f \circ g$.

such that the following axioms hold:

- associativity if $f : a \to b$, $g : b \to c$ and $h : c \to d$ then $h \circ (g \circ f) = (h \circ g) \circ f$
- identity for every object c, there exists a morphism $\mathrm{id}_c : c \to c$, such that for every $hom(a, b) \ni f$ we have: $\mathrm{id}_b \circ f = f \circ \mathrm{id}_a$=f.

Category

A category $\mathfrak{C}$ consists of

- a class $Obj(\mathfrak{C})$ of objects,
- a class $hom(\mathfrak{C})$ of morphisms between the objects. Each morphism f has a unique source object a and target object b where $a, b \in Obj(\mathfrak{C})$.
  Notation: if $f : a \to b$ then we write $f \in hom(a, b)$
- for $a, b, c \in Obj(\mathfrak{C})$, a binary operation $hom(a, b) \times hom(b, c) \to hom(a, c)$ called composition of morphisms. Notation: $f \circ g$.

such that the following axioms hold:

- associativity if $f : a \to b$, $g : b \to c$ and $h : c \to d$ then $h \circ (g \circ f) = (h \circ g) \circ f$
- identity for every object c, there exists a morphism $\mathrm{id}_c : c \to c$, such that for every $hom(a, b) \ni f$ we have: $\mathrm{id}_b \circ f = f \circ \mathrm{id}_a$=f.

Functor

Let $\mathfrak{C}$ and $\mathfrak{D}$ be categories. A covariant functor $\mathscr{F}$ from $\mathfrak{C}$ to $\mathfrak{D}$ is a mapping that:

- associates to each object $c \in Obj(\mathfrak{C})$ an object $\mathscr{F}(c) \in Obj(\mathfrak{D})$,
- associates to each morphism $hom(\mathfrak{C}) \ni f : a \to b \in$, a morphism $hom(\mathfrak{D}) \ni \mathscr{F}(f) : \mathscr{F}(a) \to \mathscr{F}(b)$

such that the following two conditions hold:

- $\mathscr{F}(\mathrm{id}_c) = \mathrm{id}_{\mathscr{F}(c)}$ for every object $c \in \mathfrak{C}$.
- $\mathscr{F}(g \circ f) = \mathscr{F}(g) \circ \mathscr{F}(f)$ for all morphisms $f : a \to b$ and $g : b \to c$.

Functor

Let $\mathfrak{C}$ and $\mathfrak{D}$ be categories. A covariant functor $\mathscr{F}$ from $\mathfrak{C}$ to $\mathfrak{D}$ is a mapping that:

- associates to each object $c \in Obj(\mathfrak{C})$ an object $\mathscr{F}(c) \in Obj(\mathfrak{D})$,
- associates to each morphism $hom(\mathfrak{C}) \ni f : a \to b \in$, a morphism $hom(\mathfrak{D}) \ni \mathscr{F}(f) : \mathscr{F}(a) \to \mathscr{F}(b)$

such that the following two conditions hold:

- $\mathscr{F}(\mathrm{id}_c) = \mathrm{id}_{\mathscr{F}(c)}$ for every object $c \in \mathfrak{C}$.
- $\mathscr{F}(g \circ f) = \mathscr{F}(g) \circ \mathscr{F}(f)$ for all morphisms $f : a \to b$ and $g : b \to c$.

Functor

Let $\mathfrak{C}$ and $\mathfrak{D}$ be categories. A covariant functor $\mathscr{F}$ from $\mathfrak{C}$ to $\mathfrak{D}$ is a mapping that:

- associates to each object $c \in Obj(\mathfrak{C})$ an object $\mathscr{F}(c) \in Obj(\mathfrak{D})$,
- associates to each morphism $hom(\mathfrak{C}) \ni f : a \to b \in$, a morphism $hom(\mathfrak{D}) \ni \mathscr{F}(f) : \mathscr{F}(a) \to \mathscr{F}(b)$

such that the following two conditions hold:

- $\mathscr{F}(\mathrm{id}_c) = \mathrm{id}_{\mathscr{F}(c)}$ for every object $c \in \mathfrak{C}$.
- $\mathscr{F}(g \circ f) = \mathscr{F}(g) \circ \mathscr{F}(f)$ for all morphisms $f : a \to b$ and $g : b \to c$.

Functor

Let $\mathfrak{C}$ and $\mathfrak{D}$ be categories. A covariant functor $\mathscr{F}$ from $\mathfrak{C}$ to $\mathfrak{D}$ is a mapping that:

- associates to each object $c \in Obj(\mathfrak{C})$ an object $\mathscr{F}(c) \in Obj(\mathfrak{D})$,
- associates to each morphism $hom(\mathfrak{C}) \ni f : a \to b \in$, a morphism $hom(\mathfrak{D}) \ni \mathscr{F}(f) : \mathscr{F}(a) \to \mathscr{F}(b)$

such that the following two conditions hold:

- $\mathscr{F}(\mathrm{id}_c) = \mathrm{id}_{\mathscr{F}(c)}$ for every object $c \in \mathfrak{C}$.
- $\mathscr{F}(g \circ f) = \mathscr{F}(g) \circ \mathscr{F}(f)$ for all morphisms $f : a \to b$ and $g : b \to c$.

Functor

Let $\mathfrak{C}$ and $\mathfrak{D}$ be categories. A contravariant functor $\mathscr{F}$ from $\mathfrak{C}$ to $\mathfrak{D}$ is a mapping that:

- associates to each object $c \in Obj(\mathfrak{C})$ an object $\mathscr{F}(c) \in Obj(\mathfrak{D})$,
- associates to each morphism $hom(\mathfrak{C}) \ni f : a \to b$, a morphism $hom(\mathfrak{D}) \ni \mathscr{F}(f) : \mathscr{F}(a) \to \mathscr{F}(b)$

such that the following two conditions hold:

- $\mathscr{F}(\mathrm{id}_c) = \mathrm{id}_{\mathscr{F}(c)}$ for every object $c \in \mathfrak{C}$.
- $\mathscr{F}(g \circ f) = \mathscr{F}(f) \circ \mathscr{F}(g)$ for all morphisms $f : a \to b$ and $g : b \to c$.

Functor

Let $\mathfrak{C}$ and $\mathfrak{D}$ be categories. A contravariant functor $\mathscr{F}$ from $\mathfrak{C}$ to $\mathfrak{D}$ is a mapping that:

- associates to each object $c \in Obj(\mathfrak{C})$ an object $\mathscr{F}(c) \in Obj(\mathfrak{D})$,
- associates to each morphism $hom(\mathfrak{C}) \ni f : a \to b$, a morphism $hom(\mathfrak{D}) \ni \mathscr{F}(f) : \mathscr{F}(a) \to \mathscr{F}(b)$

such that the following two conditions hold:

- $\mathscr{F}(\mathrm{id}_c) = \mathrm{id}_{\mathscr{F}(c)}$ for every object $c \in \mathfrak{C}$.
- $\mathscr{F}(g \circ f) = \mathscr{F}(f) \circ \mathscr{F}(g)$ for all morphisms $f : a \to b$ and $g : b \to c$.

Functor

Let $\mathfrak{C}$ and $\mathfrak{D}$ be categories. A contravariant functor $\mathscr{F}$ from $\mathfrak{C}$ to $\mathfrak{D}$ is a mapping that:

- associates to each object $c \in Obj(\mathfrak{C})$ an object $\mathscr{F}(c) \in Obj(\mathfrak{D})$,
- associates to each morphism $hom(\mathfrak{C}) \ni f : a \to b$, a morphism $hom(\mathfrak{D}) \ni \mathscr{F}(f) : \mathscr{F}(a) \to \mathscr{F}(b)$

such that the following two conditions hold:

- $\mathscr{F}(\mathrm{id}_c) = \mathrm{id}_{\mathscr{F}(c)}$ for every object $c \in \mathfrak{C}$.
- $\mathscr{F}(g \circ f) = \mathscr{F}(f) \circ \mathscr{F}(g)$ for all morphisms $f : a \to b$ and $g : b \to c$.

Functor

Let $\mathfrak{C}$ and $\mathfrak{D}$ be categories. A contravariant functor $\mathscr{F}$ from $\mathfrak{C}$ to $\mathfrak{D}$ is a mapping that:

- associates to each object $c \in Obj(\mathfrak{C})$ an object $\mathscr{F}(c) \in Obj(\mathfrak{D})$,
- associates to each morphism $hom(\mathfrak{C}) \ni f : a \to b$, a morphism $hom(\mathfrak{D}) \ni \mathscr{F}(f) : \mathscr{F}(a) \to \mathscr{F}(b)$

such that the following two conditions hold:

- $\mathscr{F}(\mathrm{id}_c) = \mathrm{id}_{\mathscr{F}(c)}$ for every object $c \in \mathfrak{C}$.
- $\mathscr{F}(g \circ f) = \mathscr{F}(f) \circ \mathscr{F}(g)$ for all morphisms $f : a \to b$ and $g : b \to c$.

Functor

Covariance:

$$\begin{array}{ccc} a & \xrightarrow{f} & b \\ \mathscr{F}\downarrow & & \downarrow\mathscr{F} \\ \mathscr{F}(a) & \xrightarrow{\mathscr{F}(f)} & \mathscr{F}(b) \end{array}$$

Natural transformation

Let $\mathscr{F}$ and $\mathscr{G}$ be functors between categories $\mathfrak{C}$ and $\mathfrak{D}$, then a natural transformation η from $\mathscr{F}$ to $\mathscr{G}$ associates to every object $a \in \mathfrak{C}$ a morphism $hom(\mathfrak{D}) \ni \eta_a : \mathscr{F}(a) \to \mathscr{G}(a)$, such that for every morphism $\mathfrak{C} \ni f : a \to b$ we have:

$$\eta_b \circ \mathscr{F}(f) = \mathscr{G}(f) \circ \eta_a$$

This equation can be expressed by the commutative diagram:

$$\begin{array}{ccc} \mathscr{F}(a) & \xrightarrow{\mathscr{F}(f)} & \mathscr{F}(b) \\ {\scriptstyle \eta_a}\downarrow & & \downarrow{\scriptstyle \eta_b} \\ \mathscr{G}(a) & \xrightarrow{\mathscr{G}(f)} & \mathscr{G}(b) \end{array}$$

Algebraic formulation

QFT on Minkowski spacetime can be formalized with the use of Haag-Kastler axioms. Main features of this framework:

- Theory formulated in terms of nets of C^*-algebras (algebras of observables) associated to subsets of Minkowski spacetime: $\mathfrak{A}(\mathcal{O})$, $\mathcal{O} \in M$.
- Physical interpretation through states (functionals on observables' algebras).
- There is a special state, that respects symmetries of M: vacuum state.

Algebraic formulation

QFT on Minkowski spacetime can be formalized with the use of Haag-Kastler axioms. Main features of this framework:

- Theory formulated in terms of nets of C^*-algebras (algebras of observables) associated to subsets of Minkowski spacetime: $\mathfrak{A}(\mathcal{O})$, $\mathcal{O} \in M$.
- Physical interpretation through states (functionals on observables' algebras).
- There is a special state, that respects symmetries of M: vacuum state.

Algebraic formulation

QFT on Minkowski spacetime can be formalized with the use of Haag-Kastler axioms. Main features of this framework:

- Theory formulated in terms of nets of C^*-algebras (algebras of observables) associated to subsets of Minkowski spacetime: $\mathfrak{A}(\mathcal{O})$, $\mathcal{O} \in M$.
- Physical interpretation through states (functionals on observables' algebras).
- There is a special state, that respects symmetries of M: vacuum state.

Algebraic formulation

QFT on Minkowski spacetime can be formalized with the use of Haag-Kastler axioms. Main features of this framework:

- Theory formulated in terms of nets of C^*-algebras (algebras of observables) associated to subsets of Minkowski spacetime: $\mathfrak{A}(\mathcal{O})$, $\mathcal{O} \in M$.
- Physical interpretation through states (functionals on observables' algebras).
- There is a special state, that respects symmetries of M: vacuum state.

Algebraic formulation

QFT on Minkowski spacetime can be formalized with the use of Haag-Kastler axioms. Main features of this framework:

- Theory formulated in terms of nets of C^*-algebras (algebras of observables) associated to subsets of Minkowski spacetime: $\mathfrak{A}(\mathcal{O})$, $\mathcal{O} \in M$.
- Physical interpretation through states (functionals on observables' algebras).
- There is a special state, that respects symmetries of M: vacuum state.

Algebraic formulation

QFT on Minkowski spacetime can be formalized with the use of Haag-Kastler axioms. Main features of this framework:

- Theory formulated in terms of nets of C^*-algebras (algebras of observables) associated to subsets of Minkowski spacetime: $\mathfrak{A}(\mathcal{O})$, $\mathcal{O} \in M$.
- Physical interpretation through states (functionals on observables' algebras).
- There is a special state, that respects symmetries of M: vacuum state.

QFT on curved spacetime

Important insights:

- Dimock: Haag-Kastler axioms for globally hyperbolic spacetimes, covariance for isometric diffeomorphisms
- Kay: Hadamard condition as a local characterization of admissible states
- Radzikowski (followed by Köhler): Hadamard condition formulated in terms of wave-front sets.
- Methods of microlocal analysis applied to QFT on curved spacetime (Fredenhagen, Brunetti, . . .)

QFT on curved spacetime

Important insights:

- Dimock: Haag-Kastler axioms for globally hyperbolic spacetimes, covariance for isometric diffeomorphisms
- Kay: Hadamard condition as a local characterization of admissible states
- Radzikowski (followed by Köhler): Hadamard condition formulated in terms of wave-front sets.
- Methods of microlocal analysis applied to QFT on curved spacetime (Fredenhagen, Brunetti, . . .)

QFT on curved spacetime

Important insights:

- Dimock: Haag-Kastler axioms for globally hyperbolic spacetimes, covariance for isometric diffeomorphisms
- Kay: Hadamard condition as a local characterization of admissible states
- Radzikowski (followed by Köhler): Hadamard condition formulated in terms of wave-front sets.
- Methods of microlocal analysis applied to QFT on curved spacetime (Fredenhagen, Brunetti, ...)

QFT on curved spacetime

Important insights:

- Dimock: Haag-Kastler axioms for globally hyperbolic spacetimes, covariance for isometric diffeomorphisms
- Kay: Hadamard condition as a local characterization of admissible states
- Radzikowski (followed by Köhler): Hadamard condition formulated in terms of wave-front sets.
- Methods of microlocal analysis applied to QFT on curved spacetime (Fredenhagen, Brunetti, . . .)

Local covariance

- Ideas developed recently by: Brunetti-Fredenhagen-Verch, Hollands-Wald.
- Application of category theory provides a formulation which doesn't fix the background
- One constructs the theory simultaneously on all spacetimes (of a given class) in a coherent way
- The theory is fixed by a covariant functor between certain categories
- Fields are natural transformations

Local covariance

- Ideas developed recently by: Brunetti-Fredenhagen-Verch, Hollands-Wald.
- Application of category theory provides a formulation which doesn't fix the background
- One constructs the theory simultaneously on all spacetimes (of a given class) in a coherent way
- The theory is fixed by a covariant functor between certain categories
- Fields are natural transformations

Local covariance

- Ideas developed recently by: Brunetti-Fredenhagen-Verch, Hollands-Wald.
- Application of category theory provides a formulation which doesn't fix the background
- One constructs the theory simultaneously on all spacetimes (of a given class) in a coherent way
- The theory is fixed by a covariant functor between certain categories
- Fields are natural transformations

Local covariance

- Ideas developed recently by: Brunetti-Fredenhagen-Verch, Hollands-Wald.
- Application of category theory provides a formulation which doesn't fix the background
- One constructs the theory simultaneously on all spacetimes (of a given class) in a coherent way
- The theory is fixed by a covariant functor between certain categories
- Fields are natural transformations

Local covariance

- Ideas developed recently by: Brunetti-Fredenhagen-Verch, Hollands-Wald.
- Application of category theory provides a formulation which doesn't fix the background
- One constructs the theory simultaneously on all spacetimes (of a given class) in a coherent way
- The theory is fixed by a covariant functor between certain categories
- Fields are natural transformations

Categories

$\mathfrak{Man}$ Obj($\mathfrak{Man}$): all four-dimensional, globally hyperbolic oriented and time-oriented spacetimes (M, g).
Morphisms: Isometric embeddings that fulfill:

- Given $(M_1, g_1), (M_2, g_2) \in$ Obj($\mathfrak{Man}$), for any causal curve $\gamma : [a, b] \to M_2$, if $\gamma(a), \gamma(b) \in \psi(M_1)$ then for all $t \in]a, b[$ we have: $\gamma(t) \in \psi(M_1)$.
- Preserving orientation and time-orientation of the embedded spacetime.

$\mathfrak{Alg}$ Obj($\mathfrak{Alg}$): C^* unital algebras
Morphisms: faithful, unit-preserving $*$-homomorphisms.

Categories

$\mathfrak{Man}$ Obj($\mathfrak{Man}$): all four-dimensional, globally hyperbolic oriented and time-oriented spacetimes (M, g).

Morphisms: Isometric embeddings that fulfill:

- Given $(M_1, g_1), (M_2, g_2) \in$ Obj($\mathfrak{Man}$), for any causal curve $\gamma : [a, b] \to M_2$, if $\gamma(a), \gamma(b) \in \psi(M_1)$ then for all $t \in]a, b[$ we have: $\gamma(t) \in \psi(M_1)$.
- Preserving orientation and time-orientation of the embedded spacetime.

$\mathfrak{Alg}$ Obj($\mathfrak{Alg}$): C^* unital algebras

Morphisms: faithful, unit-preserving $*$-homomorphisms.

Categories

$\mathfrak{Man}$ Obj($\mathfrak{Man}$): all four-dimensional, globally hyperbolic oriented and time-oriented spacetimes $(M, \boldsymbol{g})$.
Morphisms: Isometric embeddings that fulfill:

- Given $(M_1, \boldsymbol{g}_1), (M_2, \boldsymbol{g}_2) \in \text{Obj}(\mathfrak{Man})$, for any causal curve $\gamma : [a, b] \to M_2$, if $\gamma(a), \gamma(b) \in \psi(M_1)$ then for all $t \in]a, b[$ we have: $\gamma(t) \in \psi(M_1)$.
- Preserving orientation and time-orientation of the embedded spacetime.

$\mathfrak{Alg}$ Obj($\mathfrak{Alg}$): C^* unital algebras
Morphisms: faithful, unit-preserving $*$-homomorphisms.

Categories

$\mathfrak{Man}$ Obj($\mathfrak{Man}$): all four-dimensional, globally hyperbolic oriented and time-oriented spacetimes $(M, \boldsymbol{g})$.
Morphisms: Isometric embeddings that fulfill:

- Given $(M_1, \boldsymbol{g}_1), (M_2, \boldsymbol{g}_2) \in \text{Obj}(\mathfrak{Man})$, for any causal curve $\gamma : [a, b] \to M_2$, if $\gamma(a), \gamma(b) \in \psi(M_1)$ then for all $t \in]a, b[$ we have: $\gamma(t) \in \psi(M_1)$.
- Preserving orientation and time-orientation of the embedded spacetime.

$\mathfrak{Alg}$ Obj($\mathfrak{Alg}$): C^* unital algebras
Morphisms: faithful, unit-preserving $*$-homomorphisms.

Categories

$\mathfrak{Man}$ Obj($\mathfrak{Man}$): all four-dimensional, globally hyperbolic oriented and time-oriented spacetimes $(M, \boldsymbol{g})$.

Morphisms: Isometric embeddings that fulfill:

- Given $(M_1, \boldsymbol{g}_1), (M_2, \boldsymbol{g}_2) \in \mathrm{Obj}(\mathfrak{Man})$, for any causal curve $\gamma : [a, b] \to M_2$, if $\gamma(a), \gamma(b) \in \psi(M_1)$ then for all $t \in]a, b[$ we have: $\gamma(t) \in \psi(M_1)$.
- Preserving orientation and time-orientation of the embedded spacetime.

$\mathfrak{Alg}$ Obj($\mathfrak{Alg}$): C^* unital algebras

Morphisms: faithful, unit-preserving $*$-homomorphisms.

Categories

$\mathfrak{Man}$ Obj($\mathfrak{Man}$): all four-dimensional, globally hyperbolic oriented and time-oriented spacetimes $(M, \boldsymbol{g})$.
Morphisms: Isometric embeddings that fulfill:

- Given $(M_1, \boldsymbol{g}_1), (M_2, \boldsymbol{g}_2) \in \mathrm{Obj}(\mathfrak{Man})$, for any causal curve $\gamma : [a, b] \to M_2$, if $\gamma(a), \gamma(b) \in \psi(M_1)$ then for all $t \in]a, b[$ we have: $\gamma(t) \in \psi(M_1)$.
- Preserving orientation and time-orientation of the embedded spacetime.

$\mathfrak{Alg}$ Obj($\mathfrak{Alg}$): C^* unital algebras
Morphisms: faithful, unit-preserving $*$-homomorphisms.

Categories

$\mathfrak{Man}$ Obj($\mathfrak{Man}$): all four-dimensional, globally hyperbolic oriented and time-oriented spacetimes $(M, \boldsymbol{g})$.
Morphisms: Isometric embeddings that fulfill:

- Given $(M_1, \boldsymbol{g}_1), (M_2, \boldsymbol{g}_2) \in \text{Obj}(\mathfrak{Man})$, for any causal curve $\gamma : [a, b] \to M_2$, if $\gamma(a), \gamma(b) \in \psi(M_1)$ then for all $t \in]a, b[$ we have: $\gamma(t) \in \psi(M_1)$.
- Preserving orientation and time-orientation of the embedded spacetime.

$\mathfrak{Alg}$ Obj($\mathfrak{Alg}$): C^* unital algebras
Morphisms: faithful, unit-preserving $*$-homomorphisms.

Categories

$\mathfrak{Man}$ — $\mathrm{Obj}(\mathfrak{Man})$: all four-dimensional, globally hyperbolic oriented and time-oriented spacetimes $(M, \boldsymbol{g})$.
Morphisms: Isometric embeddings that fulfill:

- Given $(M_1, \boldsymbol{g}_1), (M_2, \boldsymbol{g}_2) \in \mathrm{Obj}(\mathfrak{Man})$, for any causal curve $\gamma : [a, b] \to M_2$, if $\gamma(a), \gamma(b) \in \psi(M_1)$ then for all $t \in]a, b[$ we have: $\gamma(t) \in \psi(M_1)$.
- Preserving orientation and time-orientation of the embedded spacetime.

$\mathfrak{Alg}$ — $\mathrm{Obj}(\mathfrak{Alg})$: C^* unital algebras
Morphisms: faithful, unit-preserving $*$-homomorphisms.

Locally covariant quantum field theory

A locally covariant quantum field theory is defined as a covariant functor $\mathscr{A}$ between $\mathfrak{Man}$ and $\mathfrak{Alg}$. This means that the following diagram commutes for all morphisms $\psi \in \hom_{\mathfrak{Man}}((M_1, \boldsymbol{g}_1), (M_2, \boldsymbol{g}_2))$ and all objects of $\mathfrak{Man}$:

$$\begin{array}{ccc} (M_1, \boldsymbol{g}) & \xrightarrow{\psi} & (M_2, \boldsymbol{g}') \\ \mathscr{A}\downarrow & & \downarrow\mathscr{A} \\ \mathscr{A}(M_1, \boldsymbol{g}) & \xrightarrow{\mathscr{A}(\psi)} & \mathscr{A}(M_2, \boldsymbol{g}') \end{array}$$

Denoting $\alpha_\psi \equiv \mathscr{A}(\psi)$, the covariance property reads:

$$\alpha_{\psi'} \circ \alpha_\psi = \alpha_{\psi' \circ \psi}\,, \quad \alpha_{\mathrm{id}_M} = \mathrm{id}_{\mathscr{A}(M, \boldsymbol{g})}\,,$$

for all morphisms ψ' from $\hom_{\mathfrak{Man}}((M_2, \boldsymbol{g}_2), (M_3, \boldsymbol{g}_3))$, $\psi \in \hom_{\mathfrak{Man}}((M_1, \boldsymbol{g}_1), (M_2, \boldsymbol{g}_2))$ and all objects of $\mathfrak{Man}$.

Further axioms

One can also include two further axioms which are important in QFT: causality and time-slice axiom.

- **Causality**: If there exist morphisms $\psi_j \in \hom_{\mathfrak{Man}}((M_j, \boldsymbol{g}_j), (M, \boldsymbol{g})), j = 1, 2$, such that the sets $\psi_1(M_1)$ and $\psi_2(M_2)$ are causally separated in $(M, \boldsymbol{g})$, then:

$$[\alpha_{\psi_1}(\mathscr{A}(M_1, \boldsymbol{g}_1)), \alpha_{\psi_2}(\mathscr{A}(M_2, \boldsymbol{g}_2))] = \{0\},$$

  where $[., .]$ is the commutator of given C^* algebras.

- **Time-slice axiom:** If the morphism $\psi \in \hom_{\mathfrak{Man}}((M, \boldsymbol{g}), (M', \boldsymbol{g}'))$ is such that $\psi(M)$ contains a Cauchy-surface in $(M', \boldsymbol{g}')$, then α_ψ is an isomorphism.

Further axioms

One can also include two further axioms which are important in QFT: causality and time-slice axiom.

- **Causality**: If there exist morphisms $\psi_j \in \hom_{\mathfrak{Man}}((M_j, \boldsymbol{g}_j), (M, \boldsymbol{g}))$, $j = 1, 2$, such that the sets $\psi_1(M_1)$ and $\psi_2(M_2)$ are causally separated in $(M, \boldsymbol{g})$, then:

$$[\alpha_{\psi_1}(\mathscr{A}(M_1, \boldsymbol{g}_1)), \alpha_{\psi_2}(\mathscr{A}(M_2, \boldsymbol{g}_2))] = \{0\},$$

  where $[.,.]$ is the commutator of given C^* algebras.

- **Time-slice axiom:** If the morphism $\psi \in \hom_{\mathfrak{Man}}((M, \boldsymbol{g}), (M', \boldsymbol{g}'))$ is such that $\psi(M)$ contains a Cauchy-surface in $(M', \boldsymbol{g}')$, then α_ψ is an isomorphism.

Fields

Let $\mathscr{D}$ be a functor that associates to each spacetime $\mathcal{M}$ a space of test functions $\mathscr{D}(\mathcal{M}) \in \mathrm{Obj}(\mathfrak{Test})$. A field Φ is defined as a natural transformation between $\mathscr{D}$ and $\mathscr{A}$. To any object $(M, \boldsymbol{g}) \in \mathfrak{Man}$ it associates a morphism $\Phi_{(M,g)} : \mathscr{D}(M, \boldsymbol{g}) \to \mathscr{A}(M, \boldsymbol{g})$ in such a way, that for each given isometric embedding $\chi : (M_1, \boldsymbol{g}_1) \longrightarrow (M_2, \boldsymbol{g}_2)$ following diagram commutes

$$\begin{array}{ccc} \mathscr{D}(M_1, \boldsymbol{g}_1) & \xrightarrow{\Phi_{(M_1,\boldsymbol{g}_1)}} & \mathscr{A}(M_1, \boldsymbol{g}_1) \\ \chi_* \downarrow & & \downarrow \alpha_\chi \\ \mathscr{D}(M_2, \boldsymbol{g}_2) & \xrightarrow{\Phi_{(M_2,\boldsymbol{g}_2)}} & \mathscr{A}(M_2, \boldsymbol{g}_2) \end{array}$$

where χ_* is the push forward under $\mathscr{D}$. This means:

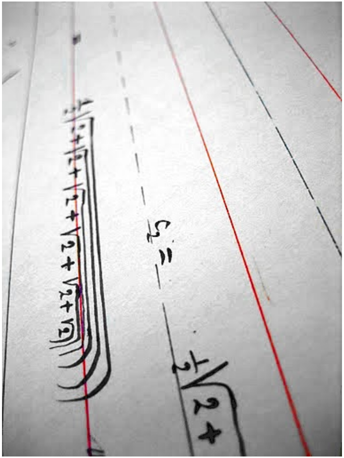
$$\alpha_\chi \circ \Phi_{(M_1,\boldsymbol{g}_1)} = \Phi_{(M_2,\boldsymbol{g}_2)} \circ \chi_* \ .$$

Perturbative quantum gravity

- splitting a metric $g_{\mu\nu}$ into background metric $\eta_{\mu\nu}$ and fluctuation $h_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- The fluctuation metric is to be quantized
- The renormalization scheme: Epstein-Glaser renormalization (valid in finite order).

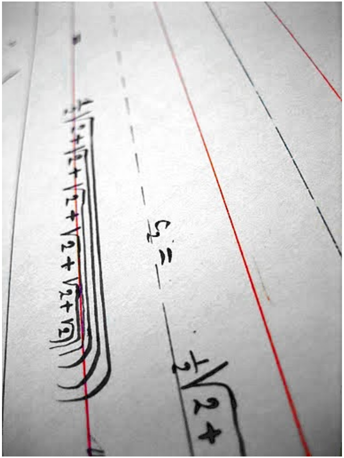


Perturbative quantum gravity

- splitting a metric $g_{\mu\nu}$ into background metric $\eta_{\mu\nu}$ and fluctuation $h_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- The fluctuation metric is to be quantized
- The renormalization scheme: Epstein-Glaser renormalization (valid in finite order).

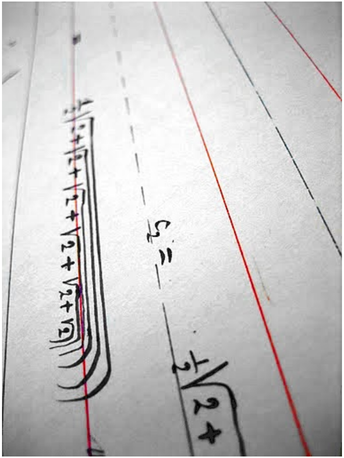


Perturbative quantum gravity

- splitting a metric $g_{\mu\nu}$ into background metric $\eta_{\mu\nu}$ and fluctuation $h_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- The fluctuation metric is to be quantized
- The renormalization scheme: Epstein-Glaser renormalization (valid in finite order).

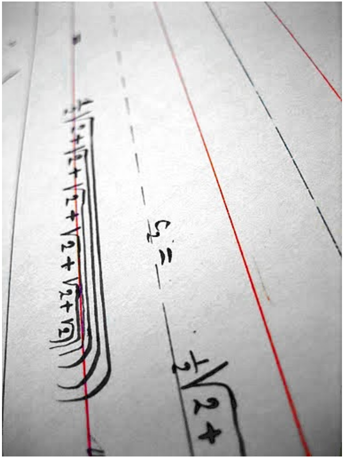


Perturbative quantum gravity

- splitting a metric $g_{\mu\nu}$ into background metric $\eta_{\mu\nu}$ and fluctuation $h_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- The fluctuation metric is to be quantized
- The renormalization scheme: Epstein-Glaser renormalization (valid in finite order).

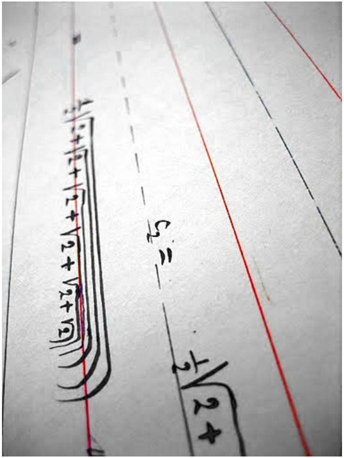


Perturbative quantum gravity

- splitting a metric $g_{\mu\nu}$ into background metric $\eta_{\mu\nu}$ and fluctuation $h_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- The fluctuation metric is to be quantized
- The renormalization scheme: Epstein-Glaser renormalization (valid in finite order).

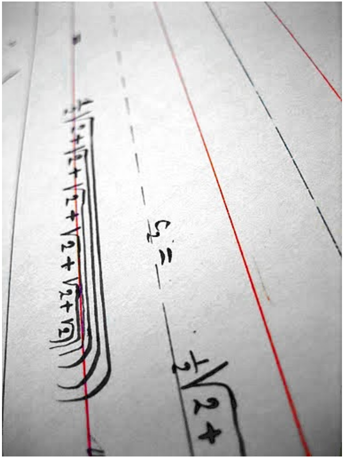


Perturbative quantum gravity

- splitting a metric $g_{\mu\nu}$ into background metric $\eta_{\mu\nu}$ and fluctuation $h_{\mu\nu}$:

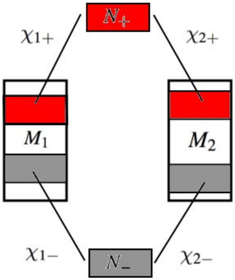
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- The fluctuation metric is to be quantized
- The renormalization scheme:
  Epstein-Glaser renormalization (valid in finite order).



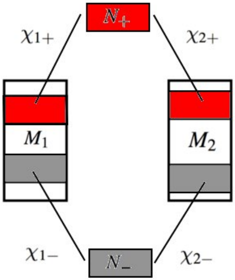
Relative Cauchy evolution

- Let N_+ and N_- be two spacetimes that embed into two other spacetimes M_1 and M_2 around Cauchy surfaces, via causal embeddings given by $\chi_{k,\pm}$, $k = 1, 2$.
- Then $\beta = \alpha_{\chi_{1+}} \alpha^{-1}_{\chi_{2+}} \alpha_{\chi_{2-}} \alpha^{-1}_{\chi_{1-}}$ is an automorphism of $\mathscr{A}(M_1)$.



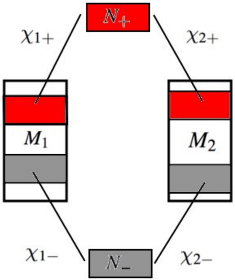
Relative Cauchy evolution

- Let N_+ and N_- be two spacetimes that embed into two other spacetimes M_1 and M_2 around Cauchy surfaces, via causal embeddings given by $\chi_{k,\pm}, k = 1, 2$.
- Then $\beta = \alpha_{\chi_{1+}} \alpha^{-1}_{\chi_{2+}} \alpha_{\chi_{2-}} \alpha^{-1}_{\chi_{1-}}$ is an automorphism of $\mathscr{A}(M_1)$.



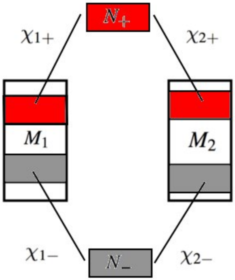
Relative Cauchy evolution

- Let N_+ and N_- be two spacetimes that embed into two other spacetimes M_1 and M_2 around Cauchy surfaces, via causal embeddings given by $\chi_{k,\pm}, k = 1, 2$.
- Then $\beta = \alpha_{\chi_{1+}} \alpha_{\chi_{2+}}^{-1} \alpha_{\chi_{2-}} \alpha_{\chi_{1-}}^{-1}$ is an automorphism of $\mathscr{A}(M_1)$.



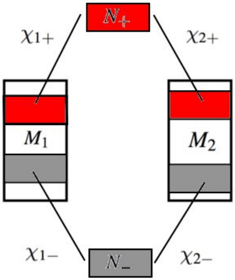
Relative Cauchy evolution

- Let N_+ and N_- be two spacetimes that embed into two other spacetimes M_1 and M_2 around Cauchy surfaces, via causal embeddings given by $\chi_{k,\pm}, k = 1, 2$.
- Then $\beta = \alpha_{\chi_{1+}} \alpha_{\chi_{2+}}^{-1} \alpha_{\chi_{2-}} \alpha_{\chi_{1-}}^{-1}$ is an automorphism of $\mathscr{A}(M_1)$.



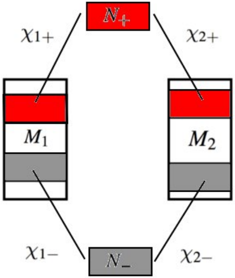
Relative Cauchy evolution

- Let N_+ and N_- be two spacetimes that embed into two other spacetimes M_1 and M_2 around Cauchy surfaces, via causal embeddings given by $\chi_{k,\pm}, k = 1, 2$.
- Then $\beta = \alpha_{\chi_{1+}} \alpha_{\chi_{2+}}^{-1} \alpha_{\chi_{2-}} \alpha_{\chi_{1-}}^{-1}$ is an automorphism of $\mathscr{A}(M_1)$.



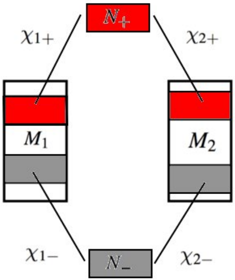
Background independence

- Let $M_1 = (M, g_1)$ and $M_2 = (M, g_2)$, where $(g_1)_{\mu\nu}$ and $(g_2)_{\mu\nu}$ differ by a (compactly supported) symmetric tensor $h_{\mu\nu}$ with $\mathrm{supp}(h) \cap J^+(N_+) \cap J^-(N_-) = \emptyset$,
- the automorphism depends only on the spacetime between the two Cauchy surfaces
- $\Theta_{\mu\nu}(x) \doteq \frac{\delta \beta_h}{\delta h_{\mu\nu}(x)}\Big|_{h=0}$ is a derivation valued distribution which is covariantly conserved
- background independance condition: $\Theta_{\mu\nu} = 0$



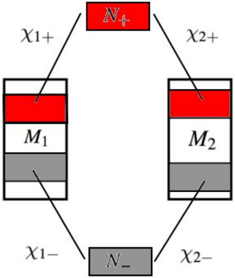
Background independence

- Let $M_1 = (M, g_1)$ and $M_2 = (M, g_2)$, where $(g_1)_{\mu\nu}$ and $(g_2)_{\mu\nu}$ differ by a (compactly supported) symmetric tensor $h_{\mu\nu}$ with $\text{supp}(h) \cap J^+(N_+) \cap J^-(N_-) = \emptyset$,
- the automorphism depends only on the spacetime between the two Cauchy surfaces
- $\Theta_{\mu\nu}(x) \doteq \frac{\delta \beta_h}{\delta h_{\mu\nu}(x)}|_{h=0}$ is a derivation valued distribution which is covariantly conserved
- background independance condition: $\Theta_{\mu\nu} = 0$



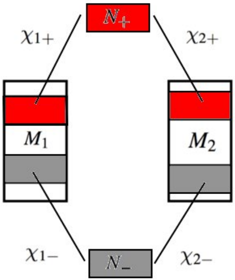
Background independence

- Let $M_1 = (M, \boldsymbol{g}_1)$ and $M_2 = (M, \boldsymbol{g}_2)$, where $(g_1)_{\mu\nu}$ and $(g_2)_{\mu\nu}$ differ by a (compactly supported) symmetric tensor $h_{\mu\nu}$ with $\mathrm{supp}(h) \cap J^+(N_+) \cap J^-(N_-) = \emptyset$,
- the automorphism depends only on the spacetime between the two Cauchy surfaces
- $\Theta_{\mu\nu}(x) \doteq \frac{\delta\beta_h}{\delta h_{\mu\nu}(x)}|_{h=0}$ is a derivation valued distribution which is covariantly conserved
- background independance condition: $\Theta_{\mu\nu} = 0$



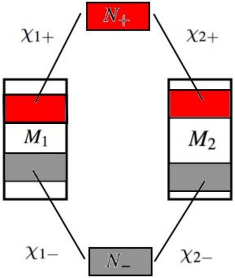
Background independence

- Let $M_1 = (M, \mathbf{g}_1)$ and $M_2 = (M, \mathbf{g}_2)$, where $(g_1)_{\mu\nu}$ and $(g_2)_{\mu\nu}$ differ by a (compactly supported) symmetric tensor $h_{\mu\nu}$ with $\mathrm{supp}(h) \cap J^+(N_+) \cap J^-(N_-) = \emptyset$,
- the automorphism depends only on the spacetime between the two Cauchy surfaces
- $\Theta_{\mu\nu}(x) \doteq \frac{\delta\beta_h}{\delta h_{\mu\nu}(x)}|_{h=0}$ is a derivation valued distribution which is covariantly conserved
- background independance condition: $\Theta_{\mu\nu} = 0$



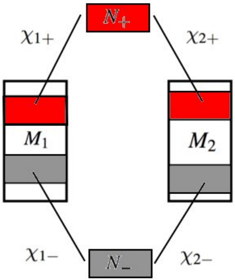
Background independence

- Let $M_1 = (M, \boldsymbol{g}_1)$ and $M_2 = (M, \boldsymbol{g}_2)$, where $(g_1)_{\mu\nu}$ and $(g_2)_{\mu\nu}$ differ by a (compactly supported) symmetric tensor $h_{\mu\nu}$ with $\mathrm{supp}(h) \cap J^+(N_+) \cap J^-(N_-) = \emptyset$,
- the automorphism depends only on the spacetime between the two Cauchy surfaces
- $\Theta_{\mu\nu}(x) \doteq \frac{\delta \beta_h}{\delta h_{\mu\nu}(x)}|_{h=0}$ is a derivation valued distribution which is covariantly conserved
- background independance condition: $\Theta_{\mu\nu} = 0$



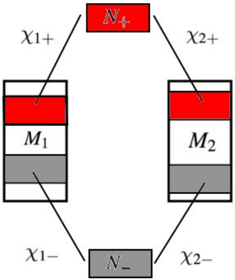
Background independence

- Let $\mathcal{M}_1 = (M, \boldsymbol{g}_1)$ and $\mathcal{M}_2 = (M, \boldsymbol{g}_2)$, where $(g_1)_{\mu\nu}$ and $(g_2)_{\mu\nu}$ differ by a (compactly supported) symmetric tensor $h_{\mu\nu}$ with $\mathrm{supp}(h) \cap J^+(\mathcal{N}_+) \cap J^-(\mathcal{N}_-) = \emptyset$,
- the automorphism depends only on the spacetime between the two Cauchy surfaces
- $\Theta_{\mu\nu}(x) \doteq \frac{\delta\beta_h}{\delta h_{\mu\nu}(x)}|_{h=0}$ is a derivation valued distribution which is covariantly conserved
- background independance condition: $\Theta_{\mu\nu} = 0$



Background independence

- Let $M_1 = (M, \boldsymbol{g}_1)$ and $M_2 = (M, \boldsymbol{g}_2)$, where $(g_1)_{\mu\nu}$ and $(g_2)_{\mu\nu}$ differ by a (compactly supported) symmetric tensor $h_{\mu\nu}$ with $\mathrm{supp}(h) \cap J^+(N_+) \cap J^-(N_-) = \emptyset$,
- the automorphism depends only on the spacetime between the two Cauchy surfaces
- $\Theta_{\mu\nu}(x) \doteq \frac{\delta\beta_h}{\delta h_{\mu\nu}(x)}|_{h=0}$ is a derivation valued distribution which is covariantly conserved
- background independance condition: $\Theta_{\mu\nu} = 0$



Problems

- Nonrenormalizability: In every order new counter terms. One has to show that theory can be applied when QG effects are small.
- Constraints have to be imposed. Best developped within perturbation theory: BRST
- BRST cohomology has to be formulated for global quantities: Fields (natural transformations)

Problems

- Nonrenormalizability: In every order new counter terms. One has to show that theory can be applied when QG effects are small.
- Constraints have to be imposed. Best developped within perturbation theory: BRST
- BRST cohomology has to be formulated for global quantities: Fields (natural transformations)

Problems

- Nonrenormalizability: In every order new counter terms. One has to show that theory can be applied when QG effects are small.
- Constraints have to be imposed. Best developped within perturbation theory: BRST
- BRST cohomology has to be formulated for global quantities: Fields (natural transformations)

Conclusions

- Locally covariant quantum field theory turned out to be successful in describing QFT on CS.
- New mathematical tools of LCQFT can be applied to perturbative quantum gravity.
- Main mathematical difficulties: BRST cohomology, renormalization procedure.
- Relations to other approaches: TQFT, perturbative QG, loops, strings …

Conclusions

- Locally covariant quantum field theory turned out to be successful in describing QFT on CS.
- New mathematical tools of LCQFT can be applied to perturbative quantum gravity.
- Main mathematical difficulties: BRST cohomology, renormalization procedure.
- Relations to other approaches: TQFT, perturbative QG, loops, strings …

Conclusions

- Locally covariant quantum field theory turned out to be successful in describing QFT on CS.
- New mathematical tools of LCQFT can be applied to perturbative quantum gravity.
- Main mathematical difficulties: BRST cohomology, renormalization procedure.
- Relations to other approaches: TQFT, perturbative QG, loops, strings ...

Conclusions

- Locally covariant quantum field theory turned out to be successful in describing QFT on CS.
- New mathematical tools of LCQFT can be applied to perturbative quantum gravity.
- Main mathematical difficulties: BRST cohomology, renormalization procedure.
- Relations to other approaches: TQFT, perturbative QG, loops, strings . . .

References I

- K. Fredenhagen, R. Brunetti, *Towards a Background Independent Formulation of Perturbative Quantum Gravity*, arXiv:gr-qc/0603079v3
- R. Brunetti, K. Fredenhagen, R. Verch, *The generally covariant locality principle - A new paradigm for local quantum field theory*, *Commun. Math. Phys.* **237** (2003) 31-68
- Haag, R., *Local Quantum Physics*, 2nd ed. Springer-Verlag, Berlin, Heidelberg, New York, 1996
- Radzikowski, M.J., *Micro-local approach to the Hadamard condition in quantum field theory in curved spacetime*, Commun. Math. Phys. **179**, 529 (1996)

References II

- R.Brunetti, K.Fredenhagen, *Microlocal analysis and interacting quantum field theories: Renormalization on physical backgrounds*, *Commun. Math. Phys.* **208**, 623 (2000)
- Segal G., *Two-dimensional conformal field theory and modular functors,Proc. IXth Intern. Congr. Math. Phys. (Bristol, Philadelphia). Eds. B.Simon, A.Truman and I.M.Davies, IOP Publ. Ltd*, 1989, 22-37

Thank you for your attention