# The BV formalism applied to classical gravity

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DER FORSCHUNG | DER LEHRE | DER BILDUNG

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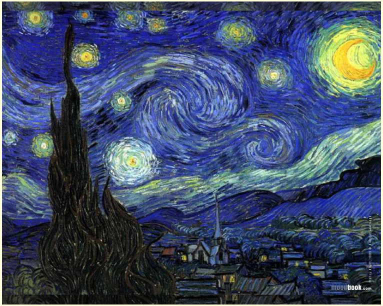
[1] based on the joint work with prof. Klaus Fredenhagen

# Based on

- K. Fredenhagen, K. R., *Batalin-Vilkovisky formalism in the functional approach to classical field theory*, [arXiv:math-ph/1101.5112].
- K. Fredenhagen, K. R., *Local covariance and background independence*, [arXiv:math-ph/1102.2376].
- R. Brunetti, K. Fredenhagen *Towards a Background Independent Formulation of Perturbative Quantum Gravity*, [arXiv:gr-qc/0603079v3].



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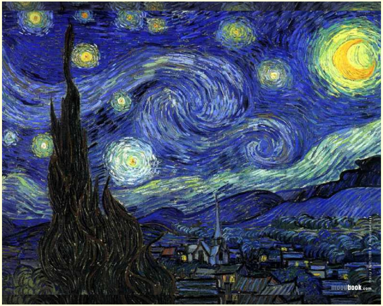

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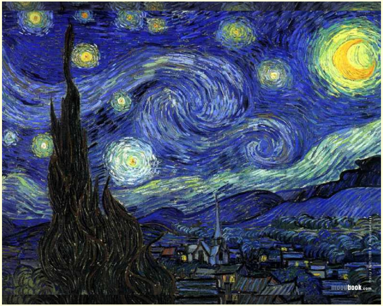

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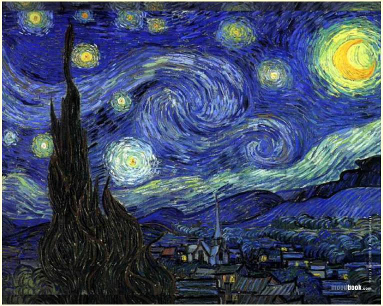

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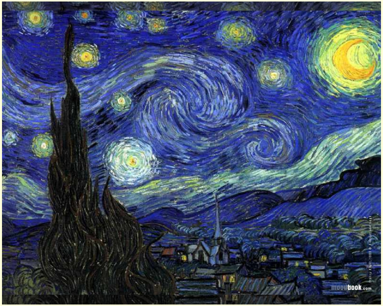

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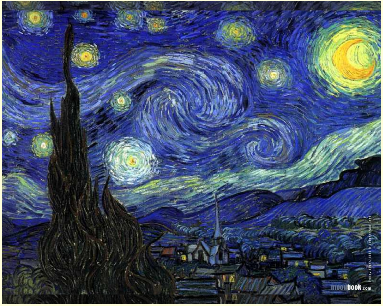

# Outline of the talk

1. Preliminaries
   - Statement of the problem
   - Equations of motion and symmetries
2. Gravity
   - Action and symmetries
   - BV construction on a fixed background
   - BV construction for natural transformations

# Statement of the problem

In our formulation with a physical system we associate:

- The configurations space $\mathfrak{E}(M)$ of all fields of the theory. $\mathfrak{E}$ is a contravariant functor from **Loc** (spacetimes) to **Vec** (lcvs).
- The space of compactly supported fields $\mathfrak{E}_c(M)$. $\mathfrak{E}_c$ is a covariant functor from **Loc** to **Vec**.
- $\mathfrak{D}$ : **Loc** $\to$ **Vec** a covariant functor that assigns to $M$ the space of compactly supported test functions $\mathfrak{D}(M)$.
- The space of smooth, compactly supported functionals on $\mathfrak{E}(M)$. This assignment also defines a covariant functor $\mathfrak{F}$ : **Loc** $\to$ **Vec** (+ regularity conditions: local, microcausal, . . . ).
- The generalized Lagrangian $L$ which is a natural transformation between functors $\mathfrak{D}$ and $\mathfrak{F}_{\text{loc}}$, s.t.: $\text{supp}(L_M(f)) \subseteq \text{supp}(f)$, and $L_M(\bullet)$ is additive in $f$. The action $S(L)$ is an equivalence class of Lagrangians. We say that $L_1 \sim L_2$ if:

$$\text{supp}(L_{1,M} - L_{2,M})(f) \subset \text{supp}\, df \quad \forall M \in \textbf{Loc},\ f \in \mathfrak{D}(M)$$

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# Local vector fields

- Vector fields $X$ on $\mathfrak{E}(M)$ (seen as a differentiable manifold) can be considered as maps from $\mathfrak{E}(M)$ to $\mathfrak{E}(M)$.
- We restrict ourselves to smooth maps $X$ with compact support and with image in $\mathfrak{E}_c(M)$ (+ regularity conditions).
- Vector fields act on $\mathfrak{F}(M)$ as derivations,

$$X(F)(\varphi) = \partial_X F(\varphi) := \langle F^{(1)}(\varphi), X(\varphi)\rangle$$

The space of such vector fields is denoted by $\mathfrak{V}(M)$. $\mathfrak{V}$ becomes a (covariant) functor by setting: $\mathfrak{V}\chi(X) = \mathfrak{E}_c\chi \circ X \circ \mathfrak{E}\chi$ .

**Antifields**

Formally we can write: $X = \int dx\, X(x) \frac{\delta}{\delta\varphi(x)}$. We can therefore identify antifields as: $\varphi^{\ddagger} = \frac{\delta}{\delta\varphi(x)}$.

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# Equations of motion and symmetries

- The EL derivative of $S$ is a natural transformation $S' : \mathfrak{E} \to \mathfrak{D}'$ defined by: $\langle S'_M(\varphi), h\rangle = \left\langle L_M(f)^{(1)}(\varphi), h\right\rangle$ with $f \equiv 1$ on supp$h$. The field equation is: $S'_M(\varphi) = 0$.
- A vector field $X \in \mathfrak{V}(M)$ is called a symmetry of the action $S$ if it holds:

$$\forall\varphi \in \mathfrak{E}(M) : \; 0 = \langle S'_M(\varphi), X(\varphi)\rangle = \partial_X(S_M)(\varphi) =: \delta_S(X)(\varphi).$$

- Space of solutions: $\mathfrak{E}_S(M) \subset \mathfrak{E}(M)$. Functionals that vanish on $\mathfrak{E}_S(M)$: $\mathfrak{F}_0(M)$. Assume that they are of the form: $\delta_S(X)$ for some $X \in \mathfrak{V}(M)$.
- Symmetries constitute the kernel of $\delta_S$.
- We obtain a resolution: $0 \to Symm. \hookrightarrow \mathfrak{V}(M) \xrightarrow{\delta_S} \mathfrak{F}(M) \to 0$.
- Functionals on $\mathfrak{E}_S(M)$: $\mathfrak{F}_S(M) \doteq \mathfrak{F}(M)/\mathfrak{F}_0(M) = H_0(\delta_S)$.
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# Action and symmetries

- The configuration space is $\mathfrak{E}(M) = (T^*M)^{\otimes 2} \doteq T^0_2 M$, the space of rank $(0,2)$ tensors.
- Let $g$ be the background metric, $h \in \mathfrak{E}(M)$ the infinitesimal perturbation and $\tilde{g} = g + h$. The Einstein-Hilbert Lagrangian reads: $L_{(M,g)}(f)(h) \doteq \int R[\tilde{g}] f \, \mathrm{d}\,\mathrm{vol}_{(M,\tilde{g})}$.
- The symmetry group is the diffeomorphism group $\mathrm{Diff}(M)$. It can be treated as an infinite dimensional Lie group modeled on $\mathfrak{X}_c(M)$, the space of compactly supported vector fields on $M$.
- The most general nontrivial symmetries can be written as elements of $\mathfrak{G}(M) := C^\infty_{\mathrm{ml}}(\mathfrak{E}(M), \mathfrak{X}_c(M))$.
- Subscript "ml" denotes the multilocal maps, i.e. algebraic completion of the space of local ones as $\mathfrak{F}(M)$-module.

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- Let $g$ be the background metric, $h \in \mathfrak{E}(M)$ the infinitesimal perturbation and $\tilde{g} = g + h$. The Einstein-Hilbert Lagrangian reads: $L_{(M,g)}(f)(h) \doteq \int R[\tilde{g}] f \, \mathrm{d}\, \mathrm{vol}_{(M,\tilde{g})}$.
- The symmetry group is the diffeomorphism group $\mathrm{Diff}(M)$. It can be treated as an infinite dimensional Lie group modeled on $\mathfrak{X}_c(M)$, the space of compactly supported vector fields on $M$.
- The most general nontrivial symmetries can be written as elements of $\mathfrak{G}(M) := \mathcal{C}^\infty_{\mathrm{ml}}(\mathfrak{E}(M), \mathfrak{X}_c(M))$.
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- The full BV complex for a fixed background reads:

$$\mathfrak{BV}(M) = \mathcal{C}^{\infty}_{\mathrm{ml}}\left(\mathfrak{E}(M), \ \bigwedge \mathfrak{E}_c(M) \ \widehat{\otimes} \ \bigwedge \mathfrak{g}'(M) \ \widehat{\otimes} \ S^{\bullet}\mathfrak{g}_c(M) \ \right)$$

- Antifields: $\#\mathrm{af} = 1$, $\#\mathrm{gh} = -1$
- Ghosts: $\#\mathrm{af} = 0$, $\#\mathrm{gh} = 1$
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# BV complex

- We expand $s$ wrt antifield number: $s = s^{(-1)} + s^{(0)}$, where:
  - $s^{(-1)}$ is the K-T differential providing the resolution of $\mathfrak{CE}_S(M)$:
    $$\ldots \to \Lambda^2\mathfrak{V} \oplus \mathfrak{G} \xrightarrow{\delta_S \oplus \rho} \mathfrak{V} \xrightarrow{\delta_S} \mathfrak{F} \to 0$$
  - $s^{(0)}$ is the Chevalley-Eilenberg differential on $\mathfrak{CE}_S(M) = C^\infty_{\mathrm{ml}}(\mathfrak{E}_S(M), \Lambda\mathfrak{g}'(M))$.
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# BV complex extended to natural transformations

The gauge invariant observables are given by:

$$H^0(\mathfrak{BV}(M), s) = H^0(\mathfrak{CE}_S(M), s^{(0)}) = \mathfrak{F}_S^{\mathrm{inv}}(M)$$

## Problem

On the fixed background the cohomology is trivial.

## Solution

We define the extended algebra of fields as: $Fld = \bigoplus_{k=0}^{\infty} \mathrm{Nat}(\mathfrak{E}_c^k, \mathfrak{BV})$.

The action of symmetries on natural transformations $\Phi \in \mathrm{Nat}(\mathfrak{E}_c, \mathfrak{F})$:

$$(\rho_M(X)\Phi_M)(f) := \partial_{\rho_M(X)}(\Phi_M(f)) + \Phi_M(\rho_M(X)f), \qquad X \in \mathfrak{X}(M)\,.$$

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- The set *Fld* becomes a graded algebra if we equip it with a graded product defined as:

$$(\Phi\Psi)_M(f_1,\ldots,f_{p+q}) = \\ = \frac{1}{p!q!}\sum_{\pi\in P_{p+q}} \Phi_M(f_{\pi(1)},\ldots,f_{\pi(p)})\Psi_M(f_{\pi(p+1)},\ldots,f_{\pi(p+q)})\,.$$

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where $s_0$ is the BV differential on the fixed background.

- The 0-cohomology of $s$ is nontrivial, since it contains for example the Riemann tensor contracted with itself, smeared with a test function:

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# Conclusions

- We gave a geometrical interpretation of the BV formalism.
- The construction was formulated in a covariant way and generalized to the natural transformations.
- In general relativity the basic physical objects are fields (natural transformations), since they are defined not on a fixed background but rather on a class of spacetimes in a coherent way.
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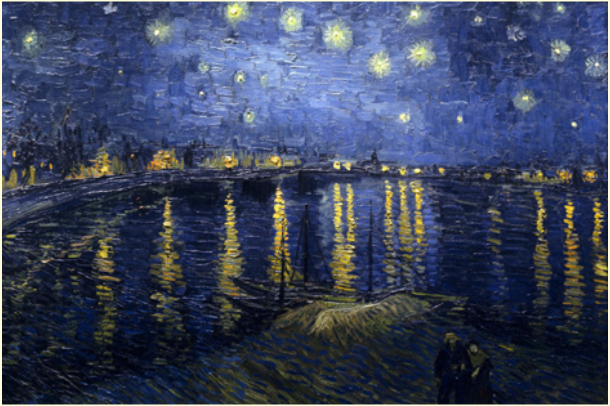

Thank you for your attention