Theories with local symmetries in the algebraic approach to perturbative quantization

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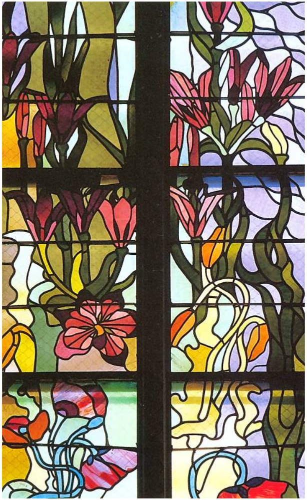
Kraków, 25.05.2012

[1] based on the joint work with Klaus Fredenhagen

Outline of the talk

BV quantization



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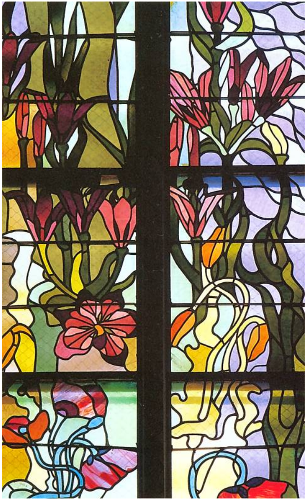

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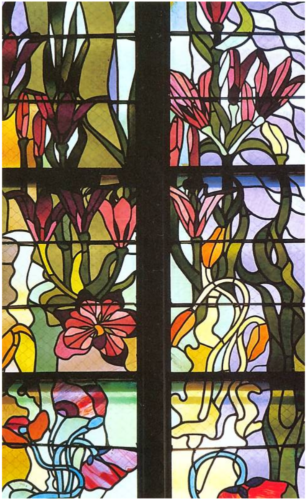

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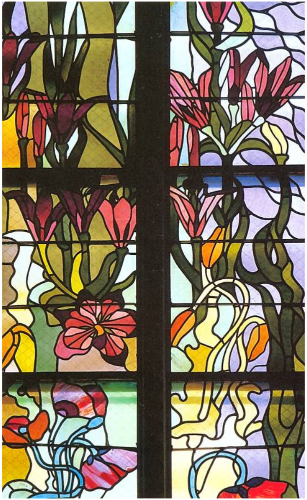

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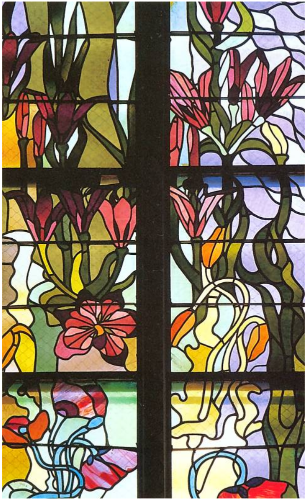

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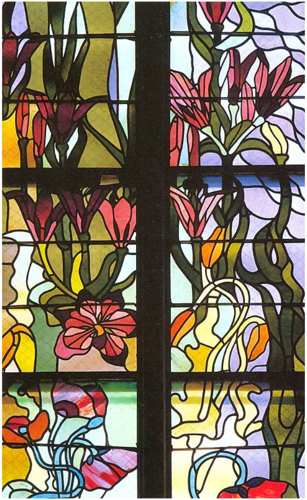

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- Quantum Master Equation (QME) used as the starting point for the construction suffers from the occurence of ill defined terms.

This talk is based on:

- K. Fredenhagen, K. R., *Batalin-Vilkovisky formalism in perturbative algebraic quantum field theory*, [arXiv:hep-th/1110.5232],

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- K. R., *Batalin-Vilkovisky formalism in locally covariant field theory*, doctoral thesis, [arXiv:hep-th/1111.5130].

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- Inductive construction the time-ordered product $\mathcal{T}^n$ of n local functionals of field configurations, requiring *causal factorization* (time ordered product is equal to the operator product if the arguments are time ordered).
- Operator product: $\star$-product, a formal deformation quantization of the Peierls bracket ([Dütsch-Fredenhagen 00, Brunetti-Fredenhagen 00, Brunetti-Dütsch-Fredenhagen 09, ...]).

Kinematical structure

- The configuration space is $\mathfrak{E}(M) = \mathcal{C}^\infty(M)$. We consider the space of smooth functionals on $\mathfrak{E}(M)$, i.e. $\mathcal{C}^\infty(\mathfrak{E}(M), \mathbb{R})$.

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$$\begin{aligned}\operatorname{supp} F = \{x \in M | \forall \text{ neighbourhoods } U \text{ of } x\ \exists \varphi, \psi \in \mathfrak{E}(M),\\ \operatorname{supp} \psi \subset U \text{ such that } F(\varphi + \psi) \neq F(\varphi)\}\ .\end{aligned}$$

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- F is local if it is of the form: $F(\varphi) = \int_M f(j_x(\varphi))\, d\mu(x)\,,$ where f is a function on the jet bundle over M and $j_x(\varphi)$ is the jet of φ at the point x.

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- Let $\mathfrak{F}(M)$ denote the space of multilocal functionals (products of local ones).

Equations of motion and symmetries

- The dynamics is given by the generalized Lagrangian which is defined as a map $\mathfrak{D}(M) \to \mathfrak{F}(M)$. For the free scalar field, i.e. $L_M(f) = \int_M \left(\frac{m^2}{2}\varphi^2 - \frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi\right) f \,\mathrm{dvol}_M$ (usual formula, but with a spacetime cutoff).

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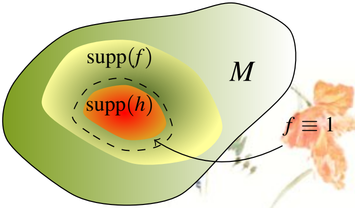

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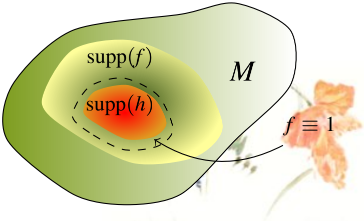
- The Euler-Lagrange derivative of S is a family of maps $S'_M : \mathfrak{E}(M) \to \mathfrak{E}'_c(M)$ defined as: $\left\langle S'_M(\varphi), h \right\rangle = \left\langle L_M(f)^{(1)}(\varphi), h \right\rangle$ with $f \equiv 1$ on supp$h$. The field equation is: $S'_M(\varphi) = 0$.



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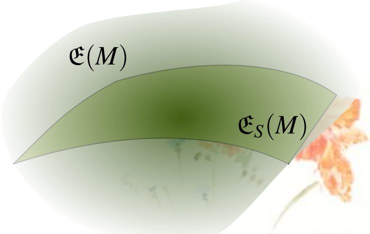

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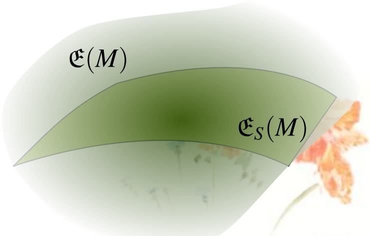
- S''_M is identified with the Klein-Gordon operator: $P = \square + m^2$.



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$$F \star G \doteq m \circ \exp(i\hbar\Gamma_\Delta)(F \otimes G)\ ,$$

where m is the pointwise multiplication and Γ_Δ is the functional differential operator

$$\Gamma_\Delta \doteq \frac{1}{2}\int dx\,dy \Delta(x,y)\frac{\delta}{\delta\varphi(x)}\otimes\frac{\delta}{\delta\varphi(y)}, \qquad \Delta = \Delta_R - \Delta_A\ ,$$

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- It holds:

$$[\Phi(f), \Phi(g)]_\star = i\hbar\langle f, \Delta g\rangle \ , \quad f, g \in \mathfrak{D}(M),$$

where $\Phi(f)(\varphi) \doteq \int f\varphi \ \mathrm{dvol}_M$ is a smeared field.

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- $\mathcal{T}$ allows us to transport the classical structure into the quantum algebra.

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- For $V \in \mathcal{T}(\mathfrak{F}_{\mathrm{reg}}(M))$ the formal S-matrix is defined as the time-ordered exponential:

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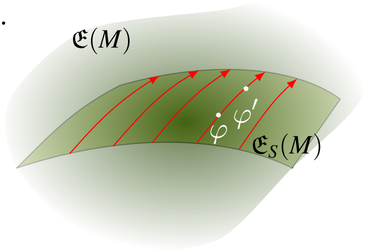
- $\mathcal{S}_V(F)$ serves as the generating functional for the interacting fields.

Symmetries

- Vector fields X on $\mathfrak{E}(M)$ can be considered as maps from $\mathfrak{E}(M)$ to $\mathfrak{E}(M)$.



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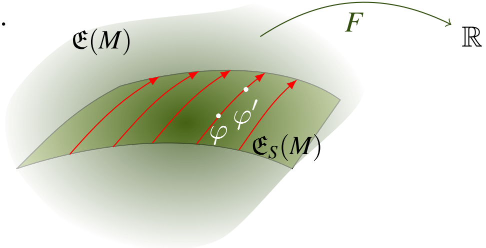

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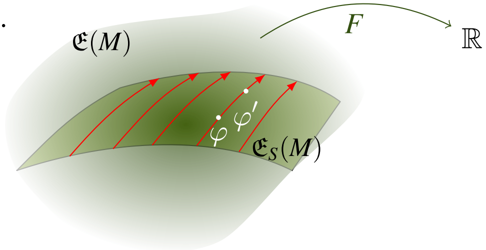

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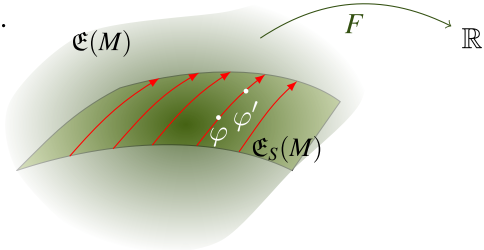

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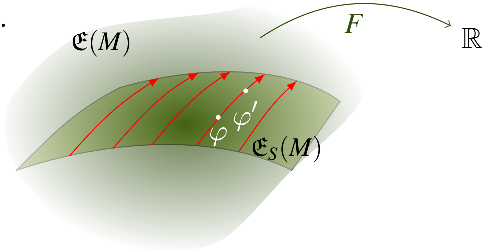

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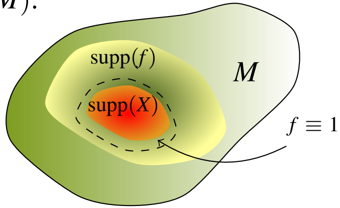

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- $X \in \mathfrak{V}(M)$ is a symmetry of S if it holds $\forall \varphi \in \mathfrak{E}(M)$:
$$0 = \langle S'_M(\varphi), X(\varphi)\rangle =: \delta_S(X)(\varphi).$$

Equations of motion and symmetries

The field equation: $S'_M(\varphi) = 0$.
A symmetry of the action: $\delta_S(X)(\varphi) = 0$.
(i.e. a direction in $\mathfrak{E}(M)$ in which the action is constant).

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- Functionals on the solution space are given by $\mathfrak{F}_S(M) = \mathfrak{F}(M)/\mathfrak{F}_0(M)$, where these in $\mathfrak{F}_0(M)$ vanish on $\mathfrak{E}_S(M)$. One can prove that elements of $\mathfrak{F}_0(M)$ are of the form: $\delta_S(X)$ for some $X \in \mathfrak{V}(M)$.

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- We obtain a sequence:

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- Functionals on $\mathfrak{E}_S(M)$:

  $$\mathfrak{F}_S(M) \doteq \mathfrak{F}(M)/\mathfrak{F}_0(M) = H_0(\delta_S).$$

Time ordering of vector fields

- The time-ordering operator on regular vector fields $\mathfrak{V}_{\mathrm{reg}}(M)$ is defined as:

$$\mathcal{T}X \doteq \int dx\, \mathcal{T}(X(x))\frac{\delta}{\delta\varphi(x)}\,,$$

where $X = \int dx\, X(x)\frac{\delta}{\delta\varphi(x)}$

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- It holds: $\partial^{\mathcal{T}}_{\mathcal{T}X} = \mathcal{T}\circ\partial_X\circ\mathcal{T}^{-1}$

Time ordered antibracket

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- We extend $\mathfrak{F}(M)$ with ghosts, antighhosts, …and their antifields (derivations of corresponding functionals), this way we obtain $\mathfrak{BV}(M)$.

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- For the moment instead of the physical interaction we take some regular functional $V \in \mathfrak{BV}_{\mathrm{reg}}(M)$ of grade $\#\mathrm{gh} = 0$.

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- Guarantees that the S-matrix on-shell doesn't depend on the gauge fixing.

Quantum BV operator

- The quantum BV operator $\hat{s}$ is defined as:

$$\hat{s}X = e_{\mathcal{T}}^{-iV/\hbar} \cdot_{\mathcal{T}} \left(\{ e_{\mathcal{T}}^{iV/\hbar} \cdot_{\mathcal{T}} X, S \}_{\star} \right) .$$

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- $\hat{s}$ on regular functionals can be also written as:

$$\hat{s} = \{ ., S + V \}_{\mathcal{T}} - i\hbar \triangle \ .$$

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- The time-ordered product $\mathcal{T}^n$ of n local functionals is well defined if their supports are disjoint.
- We can extend $\mathcal{T}^n$ to $\mathcal{T}^n_r$, which is well defined for arbitrary local functionals but the extension is not unique: renormalization ambiguity described by the renormalization group.

Associativity

Theorem (K. Fredenhagen, K.R. 2011)

The renormalized time-ordered product $\cdot_{\mathcal{T}_\mathrm{r}}$ is an associative product on $\mathcal{T}_\mathrm{r}(\mathfrak{F}(\mathbb{M}))$ given by

$$F \cdot_{\mathcal{T}_\mathrm{r}} G \doteq \mathcal{T}_\mathrm{r}(\mathcal{T}_\mathrm{r}^{-1} F \cdot \mathcal{T}_\mathrm{r}^{-1} G)\,,$$

where $\mathcal{T}_\mathrm{r} : \mathfrak{F}(\mathbb{M})[[\hbar]] \to \mathcal{T}_\mathrm{r}(\mathfrak{F}(\mathbb{M}))[[\hbar]]$ is defined as

$$\mathcal{T}_\mathrm{r} = (\oplus_n \mathcal{T}_\mathrm{r}^n) \circ \beta\,,$$

where $\beta : \mathcal{T}_\mathrm{r} : \mathfrak{F}(\mathbb{M}) \to S^\bullet \mathfrak{F}_{\mathrm{loc}}^{(0)}(\mathbb{M})$ is the inverse of multiplication m.

Renormalized QME and the quantum BV operator

- Since $\cdot_{\mathcal{T}_\mathrm{r}}$ is an associative, commutative product, we can use it in place of $\cdot_{\mathcal{T}}$ and define the renormalized QME and the quantum BV operator as:

$$\{e_{\mathcal{T}_\mathrm{r}}^{iV/\hbar}, S\}_\star = 0$$

$$\hat{s}(X) \doteq e_{\mathcal{T}_\mathrm{r}}^{-iV/\hbar} \cdot_{\mathcal{T}_\mathrm{r}} \left(\{e_{\mathcal{T}_\mathrm{r}}^{iV/\hbar} \cdot_{\mathcal{T}_\mathrm{r}} X, S\}_\star \right) ,$$

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- These formulas get even simpler if we use the anomalous Master Ward Identity ([Brenecke-Dütsch 08, Hollands 08]).

anomalous Master Ward Identity

Theorem (F. Brenecke, M. Dütsch 2008)

Let $X \in \mathcal{T}_{\mathrm{r}}(\mathfrak{V}_{\mathrm{loc}}(\mathbb{M}))$, $V, S \in \mathcal{T}_{\mathrm{r}}(\mathfrak{F}_{\mathrm{loc}}(\mathbb{M}))$, then it holds:

$$\{e_{\mathcal{T}_{\mathrm{r}}}^{iV/\hbar} \cdot_{\mathcal{T}_{\mathrm{r}}} X, S\}_\star = e_{\mathcal{T}_{\mathrm{r}}}^{iV/\hbar} \cdot_{\mathcal{T}_{\mathrm{r}}} \left(\{X, V+S\}_{\mathcal{T}_{\mathrm{r}}} - \triangle_V(X)\right),$$

where $\triangle_V(X)$ is the anomaly. It is of order $\mathcal{O}(\hbar)$ and can be written in the form $\triangle_V(X) = \sum\limits_{n=0}^{\infty} \triangle^{(n)}(V^{\otimes n}; X)$, where each $\triangle^{(n)}$ is local, linear in X and $\triangle^{(n)}(V^{\otimes n}; X) = 0$ if $\mathrm{supp} X \cap \mathrm{supp} V = \varnothing$.

Renormalized QME and the quantum BV operator

- Using the MWI we obtain following formulas:

$$0 = \frac{1}{2}\{V+S, V+S\}_{\mathcal{T}_r} - \triangle_V(V)\,,$$
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- By using the renormalized time ordered product $\cdot_{\mathcal{T}}$ we obtained in place of $\triangle$, the interaction-dependent operator $\triangle_V$.
- In the renormalized theory $\triangle_V$ is finite on local vector fields in contrast to $\triangle$.

Conclusions

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- The use of the anomalous MWI allowed us to give a simple form of the QME and the quantum BV operator,
- We showed how the ill-defined operator $\triangle$ can be replaced with the anomaly term $\triangle_V$ of the MWI.

Thank you for your attention!