Mathematics of interacting QFT models
York 1-5 July 2019

Titles and abstracts

Dorothea Bahns

The Sine Gordon model in pAQFT

Tba.

John Barrett

Bimodules and non-commutativity in fundamental physics

I explain that bimodules over non-commutative algebras give a mathematical framework for a variety of examples of geometries relevant to high energy physics and quantum gravity.

Detlev Buchholz

Classical dynamics, arrow of time, and genesis of the Heisenberg commutation relations

Based on the assumption that time evolves only in one direction and mechanical systems can be described by classical Lagrangeans on configuration space, a dynamical C*-algebra is presented for nonrelativistic particles at atomic scales. This algebra relies only on classical concepts, without presupposing any quantization scheme. But it is inherently non-commutative and comprises a large set of dynamics. The Heisenberg commutation relations for position and velocity measurements are derived from it and Hilbert space representations of the algebra lead to the conventional formalism of quantum mechanics. (Joint work with Klaus Fredenhagen).

Daniela Cadamuro

CURING THE INFRARED PROBLEM IN NONRELATIVISTIC QED

In nonrelativistic QED, the electron as an infraparticle exhibits velocity superselection, namely plane-wave configurations of the electron with different velocities give rise to inequivalent representations of the algebra of the asymptotic electromagnetic field. Moreover, as another feature of the infrared problem, the Hamiltonian has no well-defined ground state in this realm. These properties make the construction of scattering states of electrons a difficult task. In a model of one spinless electron interacting with the quantized electromagnetic field, we approach these problems in two different ways: On the one hand, by viewing the electron on a new background state, the infravacuum state, which generates a new class of representations; on the other hand, by restricting the algebra to the future light cone. In both cases, our construction leads to the absence of velocity superselection. (Joint project with W. Dybalski)

Pierre Clavier

REAL RESUMMATION IN QUANTUM FIELD THEORY

I will introduce the basic ideas of Resurgence theory, which is a vast generalisation of Borel-Laplace resummation method. Real resummations (linked to Ecalle's alien derivatives) allow to resum a divergent series in a direction where its Borel transform has singularities. We will see some implications of this concept in Quantum Field Theory.

Jan Dereziński

THE FEYNMAN PROPAGATOR ON A CURVED SPACETIME

Spacetimes with an asymptotically stationary future and past possess a distinguished Feynman propagator, a central ingredient of the evaluation of scattering amplitudes. I will describe its construction, including a discussion of various relevant tools from functional analysis.

Paweł Duch

INFRARED PROBLEM IN PERTURBATIVE QED

Because of the long-range character of interactions mediated by massless photons the standard expression for the scattering matrix in quantum electrodynamics is plagued by infrared divergences. The known pragmatic solution is to abandon the construction of the scattering matrix altogether and compute only the so-called inclusive cross sections which, however, do not provide a complete information about the scattering. In the talk, I will present a proposal for the mathematically rigorous perturbative construction of the scattering matrix in quantum electrodynamics which is proved to work at least in low orders of perturbation theory. The construction uses the adiabatic switching of the interaction and is inspired by the ideas of Dollard, Kulish and Faddeev. The constructed scattering matrix is defined in the standard Fock space and is translationally invariant. However, the physical interpretation of states and the representation of the group of spacetime translations acting in this space are non-standard.

Maximilian Duell

MULTI-PARTICLE SCATTERING AND ASYMPTOTIC COMPLETENESS IN INTERACTING WEDGE-LOCAL QFT MODELS

In this talk I will present N-particle scattering theory for wedge-local QFTs, as developed in my PhD project (Advisor: W. Dybalski). The constructive approach via the wedge-local framework was initially developed for two-dimensional integrable QFT and subsequently extended to also produce interacting models in arbitrary space-time dimension by Lechner et al. Aiming at further clarification of the physical interpretation of the various obtained (presumably non-local) models, I constructed N-particle scattering states with arbitrary particle numbers in a general massive wedge-local operator-algebraic QFT. My construction relies on wedge duality, which is very natural and was thoroughly studied in the local and wedge-local context and leads to a wedge swapping symmetry of one particle states. On these grounds I can avoid a geometric pitfall in the conventional Haag-Ruelle method requiring impossible assignments of space-like separated localizing wedges to $N>2$ particles, which has blocked progress on this topic until now. Applying my present results to massive Grosse-Lechner-type models I can confirm the persistence of interactions also beyond the two-particle sector, and the question of complete particle interpretation of these models can be posed and answered affirmatively. Thereby Grosse-Lechner-type models are exhibited as first examples of relativistic (wedge-local) QFT in four-dimensional spacetime, which are interacting and asymptotically complete.

(Partially based on CMP (2018) 364 pp.203-232)

Wojciech Dybalski

ASYMPTOTIC CHARGES, SOFT-PHOTON THEOREMS AND LARGE GAUGE TRANSFORMATIONS

In this talk we discuss rigorous results related to the Strominger's `infrared triangle'. We will discuss, in particular, the existence of asymptotic charges and soft-photon theorems in classical Maxwell-Lorentz systems. We will also derive the large gauge transformations linking the Coulomb and axial gauge in external current QED. By comparing the respective asymptotic charges we will show the unitary inequivalence of the two gauges. We will also verify that the large gauge transformation linking two axial gauges with different axes is given by the flux of the magnetic field through the region limited by the two axes.

(Based on joint works with Benedikt Wegener and Duc Viet Hoang).

David Evans

RECONSTRUCTION AND THE SEARCH FOR THE EXOTIC IN CONFORMAL FIELD THEORY

Groups act as symmetries of physical systems and on their mathematical models - such as actions on algebras of operators on Hilbert spaces. Symmetries beyond those arising from groups or their deformations as quantum groups, loop groups are feasible but hard to locate and realise. The accepted position was that the Haagerup system was exotic and surely could not be constructed from group like symmetries. We discuss work with Terry Gannon that this should be considered as misconception and the more general issue of reconstructing conformal field theories from modular tensor categories.

Chris Fewster

LOCAL MEASUREMENT SCHEMES FOR QUANTUM FIELD THEORY IN CURVED SPACETIMES

Consider an experiment in which a probe is coupled to a quantum field in a bounded spacetime region, and the probe is subsequently measured. An obvious question is how the measurement of the probe can be interpreted as an observation of the quantum field. Surprisingly, little work has been done to address it, although there is a well-developed field of quantum measurement theory, which is however rarely discussed in the context of quantum fields and still less in curved spacetimes. This talk will describe how the probe measurement can be interpreted as an observable with a defined localisation, and will discuss a number of related issues, including how a post-selected state can be defined following a selective measurement of the probe. The general formalism is illustrated with a specific model. (Joint work with Rainer Verch [Leipzig] - arXiv:1810.06512.)

Klaus Fredenhagen

CHARACTERIZATION OF DYNAMICS IN ALGEBRAIC QUANTUM FIELD THEORY

S-matrices describing external operations on a system satisfy in addition to the well-studied causal relations a further relation in terms of a classical Lagrangean. This relation is an integrated version of the Schwinger-Dyson equation. The algebra generated by these relations is a well-defined Haag-Kastler net of C*-algebras. Perturbation theory yields formal states (i.e. with values in formal power series), and in a few cases, also proper states with good continuity properties could be constructed. (Based on joint work with Detlev Buchholz and with Dorothea Bahns and Kasia Rejzner.)

Christian Gass

PERTURBATIVE CONSTRUCTION OF INTERACTIONS INVOLVING STRING-LOCAL BOSONIC POTENTIALS

Many of the issues appearing in gauge theories involving massless gauge bosons are not present when one employs alternative potentials of the field strengths which are localized along a semi-infinite ray. For example, the appearance of ghost fields and negative norm states are avoided by invoking these *string-local* potentials. Due to the weak localization, the construction of time-ordered products and the Epstein-Glaser renormalization are much more involved than in the usual approaches but there is a clear path to classifying the renormalization freedom.

Harald Grosse

SOLVING THE NONCOMMUTATIVE $\Phi^3_{2,4,6}$ QFT MODELS

Quantum field Theory led years ago to many beautiful ideas and results, but the goal to construct a mathematical consistent four-dimensional model has not been reached. In recent years, a modification of the space-time structure led to a new treatment of matrix-type models. In earlier work we used Schwinger-Dyson equations and reduced the solution of N-point functions to Riemann-Hilbert problems.

We review this procedure for the Φ^3 model in dimensions d = 2, 4, 6, perform the necessary renormalization and obtain all correlation functions explicitly. This model is closely related to the Kontsevich model, for which a correspondence between topological sectors of the moments of the measure and a procedure called topological recursion, is known.

The Schwinger-Dyson equation for the second moment corresponds to the spectral curve in topological recursion. The combinatorics of the linear integral equations for the higher moments was solved.

As a byproduct we obtained an easy algorithm to generate intersection numbers.

(Work together with Raimar Wulkenhaar [Münster], partly with Akifumi Sako [Tokyo] and partly with Alexander Hock [Münster])

Eli Hawkins

COMBINATORICS OF THE STAR PRODUCT IN AQFT

In the setting of perturbative algebraic quantum field theory, both Planck's constant and the coupling constant are treated as formal parameters. The algebra of observables is constructed by formal deformation quantization of a commutative algebra of functionals; this is expressed as a star product, i.e., an associative product on the space of formal power series with functionals as coefficients.

The important structure is the algebra (or net of algebras) rather than the precise form of the star product. The arbitrariness of the star product can be seen as the freedom to identify functionals with observables by a quantization map. We simplify the construction of quantum Møller operators significantly by a particular choice of quantization map.

The interacting star product can be constructed from the non-interacting star product by using Møller operators to identify free and interacting observables. With our preferred choice of quantization map, this calculation can be expressed diagrammatically, and leads to a simple diagrammatic formula for the interacting star product.

This formula can be resummed to a nonperturbative formula in which Planck's constant is still formal, but the coupling constant is not. This is expressed in terms of functional derivatives of the action and the retarded propagator of the linearized theory.

(Joint work with Kasia Rejzner)

Eleni Kontou

A SEMICLASSICAL SINGULARITY THEOREM

Hawking's singularity theorem concerns matter obeying the strong energy condition (SEC), which means that all observers experience a nonnegative effective energy density (EED), thereby guaranteeing the timelike convergence property. However, some classical and all quantum fields, violate the SEC. Therefore there is a need to develop theorems with weaker restrictions, namely energy conditions averaged over an entire geodesic and quantum energy inequalities (QEIs), weighted local averages of energy densities.

In this talk I will first give a derivation different from the original theorem that avoids the Raychaudhuri equation but instead makes use of index form methods. I will discuss how it improves over existing methods and how it can be applied to prove theorems with weakened hypotheses. Then I will present the derivation of a QEI for the EED and discuss how it can be used as a hypothesis to a Hawking-type singularity theorem. Finally I will present estimates of the initial contraction required at a Cauchy surface in order to guarantee future timelike geodesic incompleteness in these cases. Based on DOI: 10.1103/PhysRevD.99.045001 and a manuscript in preparation.

Gandalf Lechner

FROM THE CONSTRUCTION OF INTEGRABLE QFTS TO THE CLASSIFICATION OF UNITARY R-MATRICES

Tba.

Nguyen Viet Dang

RENORMALIZATION AND A QUESTION OF QUILLEN ON DETERMINANT LINE BUNDLES

I will discuss the problem of quantizing a quadratic Lagrangian interacting with some external potential. I will show how to make sense of the effective action by subtracting local counterterms and I shall give geometric applications related to the rigidity of Riemannian manifolds. Finally, I will relate the results to some unpublished conjecture of Quillen (30 April 1989) on determinant line bundles.

Sylvie Paycha

A GEOMETRIC PERSPECTIVE ON REGULARITY STRUCTURES

We propose an interpretation of re-expansion maps arising in regularity structures in the language of groupoids. They are viewed as direct connections on groupoids, which generalise Teleman's direct connections on morphism bundles. For gauge groupoids, built from a principal bundle, a re-expansion map can then be viewed as a (local) ``gaugeoid field'', the groupoid counterpart of a (local) gauge field. In the case of Riemannian manifolds without boundary, we compare our definition of a polynomial regularity structure with the one given by Driver, Diehl and Dahlqvist.

(This talk is based on joint work with S. Azzali, Y. Boutaïb and A. Frabetti)

Karl-Henning Rehren

SPACELIKE DEFORMATIONS

In contrast to Hamiltonian perturbation theory that deforms the time evolution of a given time-zero algebra, spacelike deformations fix the algebra of fields along the time axis and deform the spatial translations and boosts. We present examples of continuous deformations of the mass, and of discrete deformations of the helicity of massless fields.

(Joint work with V. Morinelli)

Vincent Rivasseau

QUANTUM FIELD THEORY ON RANDOM TREES

Quantum gravity leads to a statistical point of view on space and time. Random trees, often called branched polymers in physics, form the simplest interesting random space. They appear naturally as the natural scaling limit of tensor models, through the melons that dominate their 1/N expansion and are also responsible for the interesting holographic and maximally chaotic properties of Sachdev-Ye-Kitaev type models. Therefore, a systematic study of quantum field theory on random trees seems a relevant step towards quantizing gravity. We provide some first results on the behavior of Feynman amplitudes on such random trees.

Alexander Schenkel

HIGHER STRUCTURES IN ALGEBRAIC QUANTUM FIELD THEORY

Algebraic quantum field theory (AQFT) is a well-established framework to axiomatize and study quantum field theories on Lorentzian manifolds, i.e. spacetimes in the sense of Einstein’s theory of general relativity. The “traditional" AQFTs appearing in the literature are only 1-categorical algebraic structures, which turns out to be insufficient to capture the important examples given by quantum gauge theories. In this talk I will give a rather non-technical overview of our recent works towards establishing a higher categorical framework for AQFT. I will also provide a sketch how examples of such higher categorical theories can be constructed from (linear approximations of) derived stacks and how they relate to the BRST/BV formalism.

Bert Schroer

STRING-LOCAL QFT AND ITS APPLICATION TO QED

After reviewing the state of art of the recent string-local formulation of QFT involving $s \geqslant 1$ fields and illustrating some general results concerning interactions of vector mesons with lower spin matter (in particular with scalar Hermitian Higgs matter H) I turn to QED. In particular I will address those problems which were recently raised in a paper by Buchholz, Ciolli, Ruzzi and Vaselli. This includes in particular the relation between infraparticles, spacelike photon clouds and the Gauß law.

Rainer Verch

QFT IN 1+1 DE SITTER SPACETIME AND ITS MINKOWSKI SCALING LIMIT

Tba.

Fabien Vignes-Tourneret

CONSTRUCTIVE TENSOR FIELD THEORY THROUGH AN EXAMPLE

In the last ten years, a new approach to quantum gravity has emerged. Called Tensor Field Theory, it generalizes random matrix models in a straightforward way. This talk will be the occasion of recalling the main motivations for such field theories and to present the state-of-the-art of their constructive study. This is joint work with Vincent Rivasseau.

Raimar Wulkenhaar

SOLUTION OF ALL QUARTIC MATRIX MODELS

We hope to describe the exact solution of dynamic matrix models with action $S(\Phi) = \exp\big(-N\mathrm{tr}(E\Phi^2 + (\lambda/4)\Phi^4)\big)$ in the large-N limit, for *any* positive matrix E of dimension $D \coloneqq \inf\big(p\colon \mathrm{tr}\big(E^{-p/2}\big) < \infty\big) < 6$. This class of matrix models is the analogue of the Kontsevich model which will be described in Harald Grosse's talk.