EPSRC
Engineering and Physical Sciences Research Council

Modern Mathematics of Quantum Theory

York 5-7 September 2017

UNIVERSITY of York

Titles and abstracts

Dorothea Bahns

THE QUANTUM SINE-GORDON MODEL IN PAQFT

After recalling the formalism of perturbative Algebraic Quantum Field Theory (pAQFT), I will explain how it allows to prove finiteness of the S-matrix of the Sine Gordon model in different representations and how the net of local observables can be constructed.

Marco Benini

THE OPERAD OF ALGEBRAIC QUANTUM FIELD THEORY

Algebraic quantum field theory (AQFT) formalizes QFTs as functors assigning algebras (of observables) to spacetimes, subject to physical axioms such as causality. One of the questions arising in this context is the following: How can an AQFT specified only on "small regions" determine the global theory? An answer is provided by the so-called universal algebra construction due to K. Fredenhagen. This construction, however, does not a priori enforce causality at the global level. Therefore this property has to be checked case-by-case. We address this problem from a novel perspective: Starting from elementary geometric data encoding causality in a very broad sense, we construct a coloured operad whose algebras are precisely AQFTs. We use the operadic language to reinterpret the universal algebra construction and to develop a refinement that encodes causality a priori. The power of the operadic machinery provides an effective way of comparing these two constructions. This comparison leads to a model-independent geometric condition for the universal algebra to succeed in producing a causal theory. When this condition fails, the refinement of the universal algebra construction becomes crucial

Daniela Cadamuro

DIRECT CONSTRUCTION OF POINTLIKE OBSERVABLES IN THE ISING MODEL

The construction of pointlike fields in quantum integrable models was a central problem of the Form Factor Programme, which tried to achieve this by constructing their n-point functions as a series of "form factors". However, convergence questions of the series remain unresolved even in the simplest case of interacting QFT, namely the massive Ising model. This model is of interest as its classical version is related to magnetic spin chains. On the other hand, the C*-algebraic approach to the construction considers semi-local bounded operators, but yields local operators only in a very abstract way.

By combining these two approaches, we explicitly construct (all) pointlike fields of the Ising model, not in the sense of the Wightman axioms, but showing that smeared versions of the fields are closable operators affiliated with the local algebras.

Alberto Cattaneo

GEOMETRICAL CONSTRUCTION OF REDUCED PHASE SPACES

The reduced phase space of a field theory is the space of its possible initial conditions endowed with a natural symplectic structure. An alternative to Dirac's method, relying on natural geometric aspects of variational problems, was introduced by Kijowski and Tulczijev. This method also has the advantage of having a natural generalization in the BV context. In this talk, I will explain the method and describe some examples, focusing in particular on the tetradic version of general relativity in four dimensions.

Viet Nguyen Dang

SPECTRAL ANALYSIS OF MORSE-SMALE FLOWS

We discuss recent results obtained with G Rivière on the spectral theory of Morse-Smale flows on some compact manifold M. In case the Morse-Smale flow has only periodic orbits in M, then we show how our spectral analysis relates to the Reidemeister torsion which arises as a partition function of abelian BF theory. For gradient flows, we relate the Ruelle resonances of the vector field generating the flow with the asymptotic spectrum of Witten Laplacians. Then we show how our method gives some insights on a conjecture of Fukaya on Witten deformation of product structures.

Claudio Dappiaggi

On the algebraic quantization on AdS spacetime

The algebraic approach offers a mathematically rigorous and effective framework to analyse free and interacting quantum field theories on curved backgrounds. It is often assumed that the underlying spacetime ought to be globally hyperbolic, so that wave-like partial differential equations can be solved in terms of an initial value problem. Although, at first glance, such requirement appears to be rather natural, it leads to discarding several models which have attracted a lot of attention in the theoretical physics community, for example quantum field theories on asymptotically AdS spacetime. In this talk we advocate that the algebraic approach is fit to analysing also these theories and, to this end, we discuss the quantization of a free field theory on the Poincaré patch of AdS spacetime. In particular we show that, in comparison to the standard procedure, three key differences emerge. First and foremost, initial data must be supplemented with an additional, admissible boundary condition. These can be classified and, for each of these there exists a different causal propagator which can be constructed explicitly. Secondly, since AdS is a spacetime with a timelike boundary, the construction of the algebra of observables must be suitably modified, so to preserve key properties such as the time-slice axiom. Thirdly, the notion of Hadamard states is no longer valid and it must be modified so to include the possibility that singularities are reflected at the boundary of the spacetime. By constructing all AdS ground states, we can extract a generalized Hadamard condition, which we conjecture to be a natural replacement of the standard Hadamard condition on all asymptotically AdS spacetimes.

Klaus Fredenhagen

Deformation quantization of the massless free scalar field in 2 dimensions, and the massless Thirring model

Abstract: A *-product is defined on a large class of functionals of the classical scalar field, and it is shown how the algebra of the massless Thirring model emerges in this formalism.

This is joint work with Dorothea Bahns and Kasia Rejzner.

Owen Gwilliam

FACTORIZATION ALGEBRAS FROM PERTURBATIVE QUANTUM FIELD THEORY

In their work on chiral conformal field theory, Beilinson and Drinfeld introduced the notion of a factorization algebra as an algebro-geometric generalization of vertex algebras. Subsequently, this notion has evolved into forms well-suited to other kinds of geometry, as a kind of multiplicative generalization of sheaves. In joint work with Costello, we showed that a classical field theory---specified by an action functional and in Euclidean signature---always provides a commutative factorization algebra via its observables, and that a Batalin-Vilkovisky quantization of the field theory determines a noncommutative deformation of that factorization algebra of classical observables. The talk will survey this result and its consequences for specific examples, such as Chern-Simons theory and various nonlinear sigma-models.

Estanislao Herscovich

SOME MIXTURE CONDITIONS OF MONOIDAL STRUCTURES APPEARING IN QUANTUM FIELD THEORY

R. Borcherds has introduced a different point of view to formalise perturbative Quantum Field Theory (pQFT). It shares however several features with other well-known constructions in the literature (see the work by E. Stueckelberg and A. Petermann, by H. Epstein and V. Glaser, and, more recently, by R. Brunetti, K. Fredenhagen and their collaborators, and by C. Brouder, B. Fauser, A. Frabetti, R. Oeckl, to mention a few). In particular, he uses several objects which behave somehow like bialgebras and comodules over them, and which are essential in his definition of Feynman measure. The former objects don't seem however to be bialgebras in the classical sense, for their product and coproduct are with respect to two different tensor products, and similarly for comodules. Moreover, following physical motivations, these objects are given as some symmetric constructions of geometric nature.

The aim of this talk is on the one hand to show that the "bialgebras" and "comodules" introduced by Borcherds cannot "naturally" exist, and on the other side to provide a background where a modified version of the so-called "bialgebras" and "comodules" do exist. This involves a category provided with two monoidal structures satisfying some compatibility conditions. As expected, the modified version of the mentioned "bialgebras" and "comodules" are not so far from the original one, considered by Borcherds. Moreover, we remark that these new candidates allowed us to prove the main results stated by Borcherds in his article, which we shall also briefly discuss (see my manuscript "Renormalization in Quantum Field Theory", available at https://www-fourier.ujf-grenoble.fr/~eherscov).

Gandalf Lechner

YANG-BAXTER REPRESENTATIONS OF THE INFINITE SYMMETRIC GROUP

The Yang-Baxter equation (YBE) plays a prominent role in many areas of physics and mathematics, including statistical mechanics, quantum mechanics, integrable quantum field theory, quantum information theory, symmetries of categories of vector spaces, knot theory, braid groups, subfactors, and many more. Its solutions are notoriously difficult to determine in generality. Previous joint work with Alazzawi resulted in a natural equivalence relation on solutions of the YBE that is relevant in constructive algebraic QFT. In this talk, I will explain how all involutive finite-dimensional solutions of the YBE can be classified up to this equivalence, using extremal characters of the infinite symmetric group and subfactors (joint work with Pennig and Wood). A simple necessary and sufficient criterion for two solutions to be equivalent is given in terms of their partial traces. I will also explain how these structures relate to their application in integrable QFT, and the characters found in the context of DHR superselection theory.

Michael Müger

ON ORBIFOLD CONFORMAL THEORIES AND BIMODULE CATEGORIES

After a short introduction to superselection (=representation) theory of low-dimensional QFTs, I will consider finite local extensions and subtheories. I will then specialize to permutation group actions and sketch some ideas on related categorical constructions.

Nicola Pinamonti

THERMAL STATES IN PERTURBATIVE ALGEBRAIC QUANTUM FIELD THEORY: STABILITY, RELATIVE ENTROPY AND ENTROPY PRODUCTION

We analyze some properties shown by extremal KMS states for interacting massive scalar fields propagating over Minkowski spacetime, recently constructed in the framework of perturbative algebraic quantum field theories by Fredenhagen and Lindner. In particular we discuss the validity of the return to equilibrium property when the interaction Lagrangian has compact spatial support. We furthermore compute the relative entropy among these states showing that such an extent is compatible with perturbation theory.

If the adiabatic limit is considered, the return to equilibrium is in general not valid. This implies that, an equilibrium state under the adiabatic limit for a perturbative interacting theory evolved with the free dynamics does not converge to the free equilibrium state. Actually, we show that the ergodic mean of this state converges to a non-equilibrium steady state for the free theory. Finally, we analyze the entropy production in this state.

Giuseppe Ruzzi

AN ALGEBRAIC APPROACH TO THE QUANTUM ELECTROMAGNETIC FIELD

We introduce and motivate the defining properties of the universal C*-algebra of the electromagnetic quantum field.

The electromagnetic field in different physical situations can be described by regular states of the algebra. Furthermore, there regular states carrying a new kind of topological charges related to pair of generators of the algebra localized in certains spacelike but topologically non-trivial regions of the Minkowski spacetime. These charges appear also in models of non-Abelian gauge theories.

Alexander Schenkel

THE STACK OF YANG-MILLS FIELDS

I give an abstract definition and explicit construction of the stack of non-Abelian Yang-Mills fields on globally hyperbolic Lorentzian manifolds. I will show that there is a stacky analog of the Yang-Mills Cauchy problem, whose well-posedness is equivalent to a whole family of parametrized PDE problems. This talk is based on joint work with M. Benini and U. Schreiber [arXiv:1704.01378].

Christoph Schweigert

COENDS, A LEGO-TEICHMÜLLER GAME AND CORRELATORS IN (NON-)SEMISIMPLE CONFORMAL FIELD THEORY

Coends provide the appropriate categorical tool to implement a sum over all states. We use them to set up a Lego-Teichmüller game describing chiral conformal field theories based on non-semisimple modular categories. We show that bulk algebras are described by modular commutative Frobenius algebras.